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REAL ESTATE PRICE MODELING AND EMPIRICAL ANALYSIS

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ABSTRACT

Real estate is a significant aspect of asset markets, particularly in Japan, where the total monetary value of real estate amounts to two-thirds of national wealth. It comprises a similarly large fraction of the asset markets of most developed countries, indicating that the appraisal of real estate values is essential in ensuring steady and sustainable economic development. Despite the importance of real estate appraisal, real-world practices in the real estate and financial sectors are not consistent with academic theories, and even these theories fail to bridge the gap between the concepts presented in the hedonic pricing model and the concept of discounted cash flow valuation. From this point of view, our aim is to present a framework to resolve the gap between these concepts to help understand the real estate pricing both in theory and practice.

We construct a theory of real estate pricing that is directly applicable to empirical analysis. Using a dynamic portfolio optimization, we first consider as a norm a model of theoretical equilibrium prices of pieces of real estate in respect to attribute prices common to all pieces of real estate. Then, we investigate how we can extend the norm to more realistic pricing models. A logical consideration suggests the utility of introducing a mixed effect model developed with the application of the Box-Cox transformation. By using our model to analyze data obtained from Japanese Real Estate Investment Trust (J-REIT) records, we demonstrate our model's ability to some extent.

Key words: *Real estate, Attributes, Hedonic model, Mixed effect model, Box-Cox transformation.*

JEL Classification: G12, R10, C58

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REAL ESTATE PRICE MODELING AND EMPIRICAL ANALYSIS

1. INTRODUCTION

Real estate is a significant aspect of asset markets, particularly in Japan, where the total monetary value of real estate amounts to two-thirds of national wealth. It comprises a similarly large fraction of the asset markets of most developed countries, indicating that the appraisal of real estate values is essential in ensuring steady and sustainable economic development. Despite the importance of real estate appraisal, real-world practices in the real estate and financial sectors are not consistent with academic theories, and even these theories fail to bridge the gap between the concepts presented in the hedonic pricing model and the concept of discounted cash flow valuation. From this point of view, our aim is to present a framework to resolve the gap between these concepts to help understand the real estate pricing both in theory and practice.

Land and buildings have certain attributes that characterize their value in terms of their usefulness as properties and/or real estate, including floor space, age, distance from nearest subway/railway stations, etc. As these attributes provide utility to their users, they yield an economic value, which is reflected in the price of the real estate.

When a good is characterized by a combination of attributes, the economic value, or equivalently, the price of the good is described as a linear combination of the economic values of the contents of its attributes. More specifically, the price of a good is estimated to be the sum of the quantity of each attribute multiplied by its corresponding unit price. This model, referred to as the *hedonic pricing model*, was first proposed by Court (1939) for his analysis of automobile prices, and later developed by Lancaster (1966) and Rosen (1974). Although many researchers, including Epple (1987), Anderson et al. (1992), Feenstra (1995), and Ekeland et al. (2004), thereafter developed more sophisticated forms of the hedonic model, the basic concept behind their models is the same as that of Lancaster, as expressed in the following proposition:

Proposition 1.1 (Due to Lancaster, 1966)

The price of a good is a linear combination of unit prices of its attributes, in which the weights for attribute prices represent the contents of the attributes.

We refer to the property expressed in this proposition as *Lancaster's (classical) hedonic property* hereafter. This is readily applicable to statistical/empirical analyses in the form of linear regression.

Because the hedonic model accords well with the concepts underlying the field of real estate, the model framework has become relatively popular among researchers in the field. Despite the widespread application of the hedonic model to empirical analysis of real estate, there remains a gap between the theory and the practice: In theory, the model suggests statistical analyses in the form of linear regression, but in practice, the application of linear regression to data does not always yield reliable results. In such cases, researchers sometimes resort to log-linear regression to enhance the explanatory capability of the model.

Regression analysis of log-prices yields more accurate results in many cases, and is thus widely used. Other regression models, such as those that use prices or log-prices vs. log-attributes, are also popular among researchers. However, they are still expedients for empirical analysis. As described above, Lancaster's classical hedonic property states that it is prices and not log-prices that should have forms of linear combination of attribute prices. Such disparity in theory and practice indicates the existence of a research gap between the classical theory and empirical models.

The purpose of this study is to develop an advanced theory that fills this research gap and extends the concept underlying the classical hedonic model to one that is consistent with practices in empirical analysis. To this end, we consider a representative agent who plans portfolio investment strategies for tradable assets that include not only financial securities but also real estate property rights. This framework provides us with the means of extending the hedonic model by adding two factors: the introduction of the Box-Cox transformation to express possible nonlinear nature in real estate markets and the employment of the mixed effect model to address the heterogeneity in the unit prices of attributes.

The paper is organized as follows. After this brief introduction, Section 2 presents the framework of analysis. Section 3 presents a real estate pricing formula and its implications and obtains results that serve as norms for the model discussed in the sections that follow. After reconsidering the assumptions of this model, Section 4 explores a possible extension of it. Section 5 presents an empirical analysis of J-REIT data to demonstrate the functioning of the extended model. Section 6 concludes our discussion.

2. A BASIC FRAME

We consider an economy in which two types of assets are traded, financial assets N^P and real estate assets N^H, and in which economic activities are represented by the trade and consumption of a representative agent whose goal is to maximize her utility. Although real estate is a type of asset, it differs from financial assets in that merely holding the property rights of a piece of real estate does not necessarily imply that it will yield a return on investment, as it is not until a piece of real estate is utilized that it yields returns or "benefits." To clarify this matter, we propose the following assumption:

Assumption 2.1

To obtain cash flows from a piece of real estate, an owner must rent it out for the duration for which she holds the property rights to it. A lessee or tenant of the property pays the owner a rent which represents the economic value of the benefit that the tenant gains from the use of that property. The benefit derived from using this piece of real estate accrues entirely to the tenant as its user.

This assumption, in particular, the last sentence implies that for the owner to enjoy the benefits accruing from the use of a piece of real estate, she would need to rent the property to herself via a "pseudo" payment of the rent referred to as *imputed rent*. It is noteworthy that the owner and tenant are identical when we assume the existence of a representative agent in the economy. This assumption also indicates the possibility of two types of trade for a piece of real estate: trade for its property rights and trade for the right to use it by leasing the property. Thus, we hereafter consider that three types of markets exist within the economy: financial security markets, real estate property markets, and real estate lease markets.

We introduce the following notations:

$t = 0$ to ∞: Discrete timing of market trades.

$\boldsymbol{P}_t \in \mathbb{R}^{N^P}$: Financial security price vector at time *t*.

$\boldsymbol{H}_t \in \mathbb{R}^{N^H}$: Real estate price vector at time *t*.

$\boldsymbol{D}_t^P \in \mathbb{R}^{N^P}$: Vector of dividends yielded by financial securities at time *t*.

$\boldsymbol{D}_t^H \in \mathbb{R}^{N^H}$: Vector of rents paid by lessees to lessors at time *t*.

$\boldsymbol{\theta}_t \in \mathbb{R}^{N^P}$: Portfolio vector of financial security holdings at time *t*.

$\boldsymbol{\varphi}_t^+ \in \mathbb{R}^{N^H}$: Portfolio vector of real estate property right holdings at time *t*.

$\boldsymbol{\varphi}_t^- \in \mathbb{R}^{N^H}$: Portfolio vector of real estate leasing at time t, i.e., the portfolio of real estate properties currently under lease contracts at time *t*.

$\mathbf{L}_t \in \mathbb{R}^{N^H \times N^H}$: Diagonal matrix of occupancy rates of real estate at time t. (Note that $\mathbf{L}_t\boldsymbol{\varphi}_t^+$ is the occupancy adjusted supply of property right.)

Y_t: Representative agent's income at time *t*.

C_t: Representative agent's consumption at time *t*.

V_t^-: Representative agent's portfolio value before portfolio rebalancing at time *t*.

V_t: Representative agent's portfolio value after portfolio rebalancing at time *t*.

A self-financing portfolio strategy is described as follows:

$$V_t = V_t^- + Y_t - C_t - \boldsymbol{\varphi}_t^{-\prime}\boldsymbol{D}_t^H + \boldsymbol{\varphi}_t^{+\prime}\mathbf{L}_t'\boldsymbol{D}_t^H \tag{1}$$

V_t and V_{t+1}^- are represented as follows:

$$V_t = \boldsymbol{\theta}_t'\boldsymbol{P}_t + \boldsymbol{\varphi}_t^{+\prime}\boldsymbol{H}_t \tag{2}$$

$$V_{t+1}^- = \boldsymbol{\theta}_t'\left(\boldsymbol{P}_{t+1} + \boldsymbol{D}_{t+1}^P\right) + \boldsymbol{\varphi}_t^{+\prime}\boldsymbol{H}_{t+1} \tag{3}$$

Thus, a representative agent's consumption at t is given by:

$$C_t = \boldsymbol{\theta}_{t-1}'\left(\boldsymbol{P}_t + \boldsymbol{D}_t^P\right) + \boldsymbol{\varphi}_{t-1}^{+\prime}\boldsymbol{H}_t + Y_t - \boldsymbol{\varphi}_t^{-\prime}\boldsymbol{D}_t^H + \boldsymbol{\varphi}_t^{+\prime}\mathbf{L}_t'\boldsymbol{D}_t^H - \boldsymbol{\theta}_t'\boldsymbol{P}_t - \boldsymbol{\varphi}_t^{+\prime}\boldsymbol{H}_t\,. \tag{4}$$

As discussed above, real estate provides "benefits" to its users, the values of which are reflected in the rents paid by the tenants to the owners. To convert these benefits into economic values, we introduce the concept of *attributes* based on the following assumption:

Assumption 2.2

Each piece of real estate is a representation of a bundle of attributes.

We introduce the following additional notations:

K: Number of attributes.

$b_{ij,t}$: Unit content of attribute j that is contained in real estate i at time t ($j = 1,\ldots,K$, $i = 1,\ldots,N^H$). Lancaster (1966) referred this variable as *consumption technology*.

Z_{jt}: Amount of attribute j that is contained in a portfolio of real estate in use $\boldsymbol{\varphi}_t^- := \left(\varphi_{it}^-\right)_{1\le i\le N^H}$ at time t ($j = 1,\ldots,K$, $i = 1,\ldots,N^H$).

$$Z_{jt} = \sum_{i=1}^{N^H} b_{ij,t}\varphi_{it}^- \quad \text{or} \quad \boldsymbol{Z}_t = \mathbf{B}_t'\boldsymbol{\varphi}_t^- \tag{5}$$

where $\mathbf{B}_t := \left(b_{ij,t}\right)_{1\le i\le N^H;1\le j\le K} = \left(\boldsymbol{b}_{i,t}\right)_{1\le i\le N^H}$.

The representative agent wants to maximize the sum of the instantaneous utility derived during each period from the present into the infinite future. As the instantaneous utility at time t is time-additive and assumed to be a function of consumption at the time and the bundle of attributes, the objective to be maximized is defined as follows:

$$U\left(\{C_t,\ \boldsymbol{Z}_t\},\ldots,\{C_{t+\tau},\ \boldsymbol{Z}_{t+\tau}\},\ldots\right) = E_t\left[\sum_{\tau=0}^{\infty}\delta^\tau u\left(C_{t+\tau},\ \boldsymbol{Z}_{t+\tau}\right)\right] \tag{6}$$

The agent's problem is described as follows:

$$\mathbf{P}\left|\begin{array}{ll} \underset{\{\boldsymbol{\theta}_t,\boldsymbol{\varphi}_t^+,\boldsymbol{\varphi}_t^-\}}{\text{maximize}} & E_t\left[\sum_{\tau=0}^{\infty}\delta^\tau u\left(C_{t+\tau},\boldsymbol{Z}_{t+\tau}\right)\right] \\ \text{subject to} & C_{t+\tau}=\boldsymbol{\theta}'_{t+\tau-1}\left(\boldsymbol{P}_{t+\tau}+\boldsymbol{D}^P_{t+\tau}\right)+\boldsymbol{\varphi}^{+\prime}_{t+\tau-1}\boldsymbol{H}_{t+\tau}+Y_{t+\tau}-\boldsymbol{\varphi}^{-\prime}_{t+\tau}\boldsymbol{D}^H_{t+\tau} \\ & \quad +\boldsymbol{\varphi}^{+\prime}_{t+\tau}\mathbf{L}_{t+\tau}\boldsymbol{D}^H_{t+\tau}-\boldsymbol{\theta}'_{t+\tau}\boldsymbol{P}_{t+\tau}-\boldsymbol{\varphi}^{+\prime}_{t+\tau}\boldsymbol{H}_{t+\tau} \\ & \boldsymbol{Z}_{t+\tau}=\mathbf{B}'_{t+\tau}\boldsymbol{\varphi}^-_{t+\tau} \\ & \tau=0,1,\ldots \end{array}\right. \qquad (7)$$

It can clearly be observed that the problem is merely an extension of typical dynamic portfolio selection problems.

3. EQUILIBRIUM

The maximization problem (7) is solved to determine optimal portfolio strategies for financial securities, real estate property rights, and real estate leasing $\left(\boldsymbol{\theta}_t,\boldsymbol{\varphi}_t^+,\boldsymbol{\varphi}_t^-\right)$ $t=0$ to ∞. For these portfolio strategies, markets must ensure that demand is equal to supply. Without losing generality, we can assume that the total supply in each market is normalized to unity. Because we are assuming the existence of a representative agent, we can express the market clearing conditions as follows:

$$\boldsymbol{\theta}_t = \mathbf{1} \qquad (8)$$

$$\varphi_t^+ = \mathbf{1} \qquad (9)$$

$$\boldsymbol{\varphi}_t^- = \mathbf{L}_t\boldsymbol{\varphi}_t^+ \qquad (10)$$

Note that this treatment of market clearing conditions in conjunction with a dynamic portfolio optimization problem is a standard one in asset pricing theory since Lucas (1978).

The first-order necessary conditions for (7) and the market clearing conditions expressed in (8)-(10) constitute a competitive equilibrium. Based on these assumptions and conditions, the following proposition holds true:

Proposition 3.1

Let the occupancy rates $\mathbf{L}_t\ (\forall t)$ *and dividends yielded by financial securities* $\boldsymbol{D}_t^P\ (\forall t)$ *be exogenous. Within the framework and according to the assumptions described above, financial security prices, real estate prices, and real estate rents are determined by the following equations:*

$$\boldsymbol{P}_t = E_t\left[\left(\boldsymbol{P}_{t+1}+\boldsymbol{D}^P_{t+1}\right)M^C_{t+1}\right] \qquad (11)$$

$$\boldsymbol{H}_t=\mathbf{L}_t\boldsymbol{D}_t^H+E_t\left[\boldsymbol{H}_{t+1}M^C_{t+1}\right]=\mathbf{L}_t\mathbf{B}_t\hat{\boldsymbol{M}}_t^z+E_t\left[\boldsymbol{H}_{t+1}M^C_{t+1}\right] \qquad (12)$$

$$\boldsymbol{D}_t^H=\mathbf{B}_t\hat{\boldsymbol{M}}_t^z \qquad (13)$$

Where

$$M_{t+1}^{C} := \delta \frac{\partial u\left(C_{t+1}, \boldsymbol{Z}_{t+1}\right) / \partial C_{t+1}}{\partial u\left(C_{t}, \boldsymbol{Z}_{t}\right) / \partial C_{t}} \tag{14}$$

$$\hat{\boldsymbol{M}}_{t}^{Z} := \frac{\partial u\left(C_{t}, \boldsymbol{Z}_{t}\right) / \partial \boldsymbol{Z}_{t}}{\partial u\left(C_{t}, \boldsymbol{Z}_{t}\right) / \partial C_{t}} \tag{15}$$

$$C_{t} = \mathbf{1}' \boldsymbol{D}_{t}^{P} + Y_{t} \tag{16}$$

$$\boldsymbol{Z}_{t} = \mathbf{B}_{t}' \mathbf{L}_{t} \mathbf{1} \tag{17}$$

The proof is easy, and thus omitted here. □

Note that Equation (12) leads to the following:

$$\boldsymbol{H}_{t} = \mathbf{L}_{t} \boldsymbol{D}_{t}^{H} + E_{t}\left[\sum_{\tau=1}^{\infty} \mathbf{L}_{t+\tau} \boldsymbol{D}_{t+\tau}^{H} \prod_{u=1}^{\tau} M_{t+u}^{C}\right] \tag{18}$$

Equations (18) and (13) indicate that the valuation of real estate prices must follow the following steps:

(Step 1) Estimate real estate rents at each period ($\boldsymbol{D}_{t}^{H}$) by converting the contents of the attributes of a piece of real estate ($\mathbf{B}_{t}$) into monetary values. The multipliers employed for the conversion are the marginal rates of substitution between the attributes and consumption ($\hat{\boldsymbol{M}}_{t}^{Z}$), and hence can be regarded as the unit prices of attributes.

(Step 2) Estimate the occupancy-adjusted rent payment at each period by calculating $\mathbf{L}_{t} \boldsymbol{D}_{t}^{H}$.

(Step 3) Calculate the discounted sum of the occupancy rate-adjusted rent payment employing the discount factors of the products of marginal rates of substitution between consumption during two consecutive periods (M_{t}^{C}).

Note that the above steps are consistent with a typical discounted-cash flow (DCF) valuation technique and that Step 1 reflects Lancaster's classical hedonic pricing theory as expressed in Proposition 1.1. It should also be noted that this hedonic pricing model applies to real estate *rents*, but does not necessarily also apply to real estate *prices*, a consideration that we examine in detail below.

Using (13) and (14), from (18) we obtain the following:

$$H_{i,t} = E_{t}\left[\sum_{\tau=0}^{\infty} \delta^{\tau} L_{i,t+\tau} \boldsymbol{b}_{i,t+\tau} \boldsymbol{M}_{t+\tau}^{Z}\right] \quad (i = 1, \ldots, N^{H}) \tag{19}$$

where $\boldsymbol{M}_{t+\tau}^{Z} := \frac{\partial u\left(C_{t+\tau}, \boldsymbol{Z}_{t+\tau}\right) / \partial \boldsymbol{Z}_{t+\tau}}{\partial u\left(C_{t}, \boldsymbol{Z}_{t}\right) / \partial C_{t}}$.

For real estate prices to reflect Lancaster's classical hedonic property as expressed in Proposition 1.1, they must be able to be expressed in the following form:

$$H_i = \boldsymbol{b}_i \boldsymbol{\mu} \quad \text{or} \quad H_i = \sum_{j=1}^{K} b_{ij} \mu_j \tag{20}$$

where $\boldsymbol{\mu}$ represents the unit prices of the attributes and is independent of individual real estate i. It is apparent that this is not applicable to Equation (19), and which thus does not reflect the hedonic property of Proposition 1.1. For Equation (19) to reflect the hedonic property, we must introduce another assumption and define a condition as below.

Assumption 3.1

The contents of the attributes contained in each piece of real estate remain constant over time such that

$$\boldsymbol{b}_{i,t} = \boldsymbol{b}_i \quad (\forall i, t) \tag{21}$$

This assumption accounts for many attributes, including floor space in square meters and walking distance from the nearest subway/railway stations. However, for some attributes, such as age of building or house in years, the assumption may not apply, and thus its application may be limited. Despite this limitation, application of this assumption yields insightful results when we introduce the following condition:

Condition 3.1

Each exogenous occupancy rate at time t ($L_{i,t}$) is independent of either the individual piece of real estate i or time t such that $L_{i,t}$ remains uniform or constant, i.e., $L_{i,t} \equiv L_t$ or $L_{i,t} \equiv L_i$.

This condition is quite restrictive, and even unrealistic. The occupancy rate (or, equivalently, 1 – the vacancy rate) is determined by economic conditions outside our framework, based on our assumption of the existence of exogenous occupancy rates as was assumed in Proposition 3.1. Depending on the overall economic situation as well as that of the area surrounding a particular piece of real estate, occupancy rates may fluctuate. As such, they may be heterogeneous, a consideration that we discuss in more detail in a later section. Further consideration of Condition 3.1 leads to the following proposition:

Proposition 3.2

Assuming the above competitive equilibrium setting and assumptions, real estate prices always reflect Lancaster's classical hedonic property as expressed in Proposition 1.1 if and only if Condition 3.1 holds true.

Proof:

Under Assumption 3.1, Equation (19) is equivalent to the following:

$$H_{i,t} = \boldsymbol{b}_i E_t \left[\sum_{\tau=0}^{\infty} \delta^{\tau} L_{i,t+\tau} \boldsymbol{M}_{t+\tau}^{Z} \right] = \boldsymbol{b}_i \boldsymbol{\pi}_{i,t} \tag{22}$$

where $\boldsymbol{\pi}_{i,t} := E_t \left[\sum_{\tau=0}^{\infty} \delta^{\tau} L_{i,t+\tau} \boldsymbol{M}_{t+\tau}^{Z} \right]$.

When $L_{i,t}$ is independent of i, (22) is expressed in the following form:

$$H_{i,t} = \boldsymbol{b}_i \hat{\boldsymbol{\pi}}_t \tag{23}$$

where $\hat{\boldsymbol{\pi}}_t := E_t \left[\sum_{\tau=0}^{\infty} \delta^{\tau} L_{t+\tau} \boldsymbol{M}_{t+\tau}^{Z} \right]$.

Similarly, when $L_{i,t}$ is independent of t, (22) is expressed in the following form:

$$H_{i,t} = L_i \boldsymbol{b}_i \hat{\hat{\boldsymbol{\pi}}}_t \tag{24}$$

where $\hat{\hat{\boldsymbol{\pi}}}_t := E_t \left[\sum_{\tau=0}^{\infty} \delta^{\tau} \boldsymbol{M}_{t+\tau}^{Z} \right]$.

Expressions (23) and (24) indicate that Equation (19) results in the form of (20), which completes the proof. □

4. EXTENSION

In the previous section, we developed a dynamic equilibrium model by combining Lancaster's classical hedonic pricing model with DCF methodology for the purpose of real estate valuation. Proposition 3.1 indicates that while the classical hedonic property expressed in Proposition 1.1 always holds true for real estate *rents*, it does not always hold true for real estate *prices*. Proposition 3.2 described the settings and conditions necessary for the hedonic property in real estate prices to hold true. Our discussion so far thus yields the following implication:

Implication of Proposition 3.2

The classical hedonic property expressed in Proposition 1.1 for real estate prices holds true entirely on the basis of the following conditions:

1. Competitive equilibrium as is described in Proposition 3.1, on the basis of underlying assumptions and conditions, always holds true for real estate markets.

2. Occupancy rates ($L_{i,t}$) are constant over time t or are uniform with respect to an individual piece of real estate i (Condition 3.1).

If we relax the assumptions that construct Proposition 3.1, the classical hedonic pricing model for real estate prices is modified naturally and this provides us a means of extending the model to develop a more advanced form that is useful in empirical analysis.

Again, the classical hedonic property expressed in Proposition 1.1 reflects the fact that real estate prices are represented by a linear functional form of (20). This suggests that a simple regression model to be applied in empirical analysis has the following form:

$$H_i = \boldsymbol{x}_i\boldsymbol{\theta} + \varepsilon_i \quad (i = 1, \ldots, N^H) \tag{25}$$

where N^H is the number of pieces of real estate, $\boldsymbol{x}_i = \left(1\,\boldsymbol{b}_i\right) \in \mathbb{R}^{1\times(1+K)}$ is the factor of the attribute contents, $\boldsymbol{\theta} \in \mathbb{R}^{(1+K)\times 1}$ is the regression coefficient, and $\epsilon_i \sim \mathcal{N}_1\left(0, \sigma_\varepsilon^2\right)$ is the error term. However, as we discussed above, if the market is not based on the assumptions that construct Proposition 3.1, we must abandon our use of linear functional forms. One of the several means of representing nonlinearity is application of the Box-Cox transformation (Box and Cox, 1964). By its application, the left side of (25) is replaced by the following:

$$H_i^*(\lambda) := \begin{cases} \dfrac{H_i^\lambda - 1}{\lambda} & (\text{if } \lambda \neq 0) \\ \log H_i & (\text{if } \lambda = 0) \end{cases} \tag{26}$$

Note that the parameter λ being equalized to unity indicates that $H_i^*\left(\lambda\right)$ essentially assumes the same value as does H_i (less unity). Thus, we can consider the divergence of λ from unity to be a measure of the degree of nonlinearity (or divergence from a linear functional form of (20)). To estimate a value of λ that fits real data, we can consider the following regression model instead of (25).

$$H_i^*\left(\lambda\right) = \boldsymbol{x}_i\boldsymbol{\theta} + \varepsilon_i \quad (i = 1, \ldots, N^H) \tag{27}$$

To advance our model development, we examined the second condition, Condition 3.1, which, as we discussed in the previous section, is quite restrictive. When we remove this condition, the classical hedonic property expressed in Proposition 1.1 does not hold for real estate prices, in which case Equation (22) holds at most under Assumption 3.1. In this equation, $\boldsymbol{\pi}_{i,t}$ represents a set of the unit prices of the attributes at t that varies depending on individual pieces of real estate. To incorporate such individuality into regression models, we replace $\boldsymbol{\theta}$ in (27) with $\boldsymbol{\theta}_i$ to allow regression coefficients to randomly vary depending on individual pieces of real estate. With such replacement, $\boldsymbol{\theta}_i$ can be interpreted as $\boldsymbol{\pi}_{i,t}$ of Equation (22) at each *t*. Thus, we can assume that $\boldsymbol{\theta}_i$ consists of two parts: a common unit price of the attributes shared by all pieces of real estate and an individual unit price which may vary randomly for each piece of real estate as follows:

$$\boldsymbol{\theta}_i = \boldsymbol{\beta} + \boldsymbol{\nu}_i \tag{28}$$

We can then introduce the following regression model to yield more realistic real estate prices instead of (27):

$$H_i^*(\lambda) = \boldsymbol{x}_i(\boldsymbol{\beta} + \boldsymbol{\nu}_i) + \varepsilon_i \tag{29}$$

To define a more specific form of regression, we introduced the concepts of *stratified sampling* to divide pieces of real estate into homogeneous strata based on such factors as geographical location features. By letting $C_l \quad (l = 1, \ldots, N)$ denote *stratum*, we can express (28) in the following specific form:

$$\boldsymbol{\theta}_i = \boldsymbol{\beta} + \sum_{l=1}^{N} \boldsymbol{\nu}_l \mathbf{1}_{i \in C_l} \tag{30}$$

where $\mathbf{1}_{i \in C_l} := \begin{cases} 1 & (\text{if } i \in C_l) \\ 0 & (\text{otherwise}) \end{cases}$. The model (29) is revised to the following form:

$$H_{li}^*(\lambda) = \boldsymbol{x}_{li}(\boldsymbol{\beta} + \boldsymbol{\nu}_l) + \varepsilon_{li} \quad (\text{if } i \in C_l) \tag{31}$$

Here we assume that $\boldsymbol{\nu}_l \sim \mathcal{N}_{1+K}(\mathbf{0}, \mathbf{G})$ where K is the number of attributes, $\mathbf{G}$ is simply assumed to be a diagonal matrix, and ε_{li} is an i.i.d. error term with $\varepsilon_{li} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$.

The concept of randomly varying regression coefficients is contained within *random coefficient models* or more generally *mixed effect models*, whose application is becoming more widespread in the analysis of longitudinal and/or panel data (Hsiao, 2003; Fitzmaurice et al., 2004; McCulloch et al., 2008).

5. EMPIRICAL ANALYSIS

In this section, we demonstrate how our model ((31) with (26)) can be applied to the analysis of buildings and houses in real estate markets. More specifically, we conduct an empirical analysis on prices in the real estate market in two viewpoints. First, we measure the degree of non-linearity by estimating λ in (26). Second, we examine if incorporating the individuality in regression coefficients, which are interpreted as attribute prices, will yield the better explanations for market prices. This is implemented by comparing mixed and fixed effect modeling on independent variables of attributes.

We derived the data set used for the analysis from J-REIT records for the years 2005–2010. This data set comprises transaction buying/selling prices and attributes for each piece of real estate. Notice that these pieces of real estate are presumed to construct the J-REITs' portfolios. The fact is consistent with our definition of *strata*: As mentioned previously, we defined *strata* for buildings and houses. The real estate described in J-REIT records can be categorized by its use as well as the district where it is located. Thus, this stratification can be justified in the aspect of J-REITs' portfolio strategies in which the use and district are their primal interests.

Regarding uses, we specified the four strata of *office, commercial, residential,* and *other uses*. It is reasonable to presume that these uses are clearly differentiated from each other while individual pieces of real estate still share common attributes across different strata. Thus, we naturally assume that on a set of common attributes, there are different prices that reflect the distinctions of strata. It is our intention that empirical analyses with the model of (28) will justify such assumption.

Per districts, most pieces of real estate owned by J-REITs are concentrated to central wards of Tokyo. The practical definition for the central wards of Tokyo is usually three wards of Chiyoda, Chuo and Minato or five wards that include Sinjuku and Shibuya in addition to the three wards. We then find that the definition of three wards can divide the pieces of real estate as equally as possible throughout the year and among strata. Hence regarding district, we specified two strata of the *three central wards of Tokyo* (*Tokyo 3*) and *all remaining districts*. Thus as a total, we yield eight strata. Note that due to the limitation of the J-REITs' data, we will not be able to introduce more sophisticated strata: Beyond this stratification, we may seem to cause more inaccurate estimation result than the one shown below.

Among the many possible attributes that we could have specified, we confined our focus to the following basic attributes: floor space in square meters ($x^{(SQMT)}$), age of building or house in years ($x^{(AGE)}$), and walking distance from nearest subway/railway station as a measure of locational convenience ($x^{(WALK)}$). We considered other attributes as residual factors.

A mixed effect model for the J-REIT record according to the above specifications is expressed as follows:

$$H_{lm}^{*}(\lambda) =$$
$$\sum_{k=1}^{8} x_{lm}^{(k)} \beta^{(k)} + x_{lm}^{(SQMT)} \left(\beta^{(SQMT)} + \nu_l^{(SQMT)}\right) + x_{lm}^{(AGE)} \left(\beta^{(AGE)} + \nu_l^{(AGE)}\right) + x_{lm}^{(WALK)} \left(\beta^{(WALK)} + \nu_l^{(WALK)}\right) + \varepsilon_{lm}$$
$$\left(\forall m \in Stratum\ l, \forall l\right) \tag{32}$$

and $x_{lm}^{(k)} = \begin{cases} 1 & (\text{if } k = l) \\ 0 & (\text{if } k \neq l) \end{cases}$ is a dummy variable representing the intercept for each stratum.

Here $l\left(1,\ldots,8\right)$ specifies *stratum* and $m\left(1,\ldots,n_l\right)$ represents the m^{th} piece of real estate in each stratum. Note that n_l is the number of pieces of real estate in stratum l and that $\sum_{l=1}^{8} n_l = N^H$.

For the purpose of comparison, we also consider a model without random coefficients, which is expressed as follows:

$$H_m^*(\lambda) = \sum_{k=1}^{8} x_{lm}^{(k)} \beta^{(k)} + x_m^{(SQMT)} \beta^{(SQMT)} + x_m^{(AGE)} \beta^{(AGE)} + x_m^{(WALK)} \beta^{(WALK)} + \varepsilon_m \quad (\forall m \in Stratum\ l, \forall l) \tag{33}$$

We hereafter refer to this model as the *fixed effect model*. The right side of the model is a simple regression model with a dummy variable representing the intercept for each stratum.

Tables 1 and 2 summarize the J-REIT data to be analyzed. Table 1 shows the geometric means of building/housing prices (in hundred million yen) and the number of available records in parentheses beneath each stratum for each year. The last column on the right shows cross-section averages while the last row on the bottom shows time-series averages. Table 2 shows the arithmetic means of the contents of attributes (in square meters, years, or minutes) for each stratum. Again, the last column on the right shows cross-section averages.

To estimate models (32) and (33), we used the MIXED procedure of *SAS 9.1.3* (Littell et al., 2006). The estimations of the model are obtained by the use of the restricted maximum likelihood (REML) method, which yields the best linear unbiased predictions (BLUP). To obtain estimations of the Box-Cox transformation parameter λ, we used the technique proposed by Gurka et al. (2006).

The estimation results for the years 2005–2010 are shown in Tables 3 to 10, showing the results for the mixed and fixed effect models of (32) and (33), respectively.

<u>Model fitness</u>

We measured the relative degrees of fitness of our models by obtaining their Akaike Information Criteria (AIC) values, which are shown in Table 3. A lower AIC value indicates a higher degree of fitness. By comparing the AIC values for mixed effect models with those for fixed effect models, we could verify that the former models yield more accurate results than the latter for all years.

At the same Table 3, we describe our estimation of the Box-Cox transformation parameter λ. As we discussed in Section 4, λ is a measure of the degree of nonlinearity. λ set at unity indicates that a simple linear regression of (25) works well. As all the results shown in the tables diverge from unity, they reject the effectiveness of a simple linear regression.

Note that if λ is set equal to null, (26) results in the logarithm of H_i. As we discussed in the introduction, performing regressions of log-prices is a widespread practice. However, the estimation results shown in Tables 3 indicate that such a practice is not justified.

We then proceed to Tables 5-10 which show estimation results by fixed and mixed effect models for each year in 2005-2010. Although they are slightly different in value, estimated fixed coefficients both for fixed and mixed effect models behaves quite similarly for each year. We then take a close look at the estimation results for each of attributes and intercepts.

Attribute: Floor space in square meters

For the fixed effect model, we found that *all* the estimations that we obtained for the attribute of floor space were significant. For the mixed effect model, we found that *most* estimations, in particular those for the strata of office and residential use and the stratum of commercial use in areas other than Tokyo 3, were significant.

Attribute: Age in years

For the fixed effect model, we found that *all* the estimated coefficients of the attribute of age were significant. For the mixed effect model, we found that *most* of the estimated coefficients of the attribute of age were *not* significant, with the stratum of residential use in areas other than Tokyo 3 being an exception, as we found its estimated coefficient to be significant for the mixed effect model. Note that that estimated coefficients have a negative value due to the fact that increases in this attribute lead to decline in prices.

Attribute: Walking distance from nearest subway/railway station

For both the fixed and mixed effect models, we found that most of the estimated coefficients of walking distance from nearest subway/railway stations were not significant. One exception was the stratum of residential use in areas other than Tokyo 3 for the mixed effect model, which we found to yield several significant results.

Intercept: Stratum dummy

At Table 4, almost all the intercepts are estimated significantly both for fixed and mixed effect models. Exceptions are the results for the stratum of other use on 2009-2010, since there were no data as shown in Table 1. Since we modeled these intercepts as stratum dummy variables as (32) and (33), we can regard the stratification by uses and districts as the important attributes for real estate pricing.

As a last remark, let us discuss the accuracy about our estimation results. At least, there exist two possible sources that bring inaccuracy to our estimation results. Since we have limited our analysis to J-REITs data, data size cannot be large enough to analyze the whole real estate market via the stratification we employed here. Especially for the strata of commercial and other use, data size is relatively small. This may cause inaccuracy in estimation results for those two strata. Moreover we conjecture that there is a sample bias in J-REITs data. It is due to two reasons: One reason is that J-REITs contain investment-purpose pieces of real estate only. Another reason is that there were few transactions in 2008 and 2009 due to the Lehmann shock which can be seen in Table 1. The issue of inaccuracy is left as another future research direction.

6. CONCLUSION

We constructed a theory of real estate pricing readily applicable to empirical analysis. We consider *attributes* that characterize pieces of real estate including floor space in square meters, age of building or house in years, and walking distance from nearest subway/railway stations as a measure of locational convenience, etc. Using a dynamic portfolio optimization, we demonstrated that in situations under defined technical conditions, the theoretical equilibrium price of a piece of real estate can be described as a linear combination of attribute prices common to all pieces of real estate. However, in the absence of such technical conditions i.e., under more realistic situations, real estate prices may diverge from their theoretical equilibrium prices, which leads to our consideration of more realistic models.

The model we propose is an extension of a classical hedonic model that accommodates real estate prices and applies sophisticated statistical techniques, including the use of a mixed effect feature and the application of the Box-Cox transformation. The mixed effect feature reflects the individuality of unit prices of attributes. The Box-Cox transformation reflects nonlinear nature of market prices. By applying our model to the analysis of J-REIT data, we could verify that it yields more accurate results compared to those obtained using fixed effect models.

REFERENCES

Andersen, S.P., de Palma, A. & Thisse, J-F. (1992). *Discrete choice theory of product differentiation*. (Cambridge: MIT Press)

Box, G.E.P. & Cox, D.R. (1964). An analysis of transformations (with discussion). *Journal of the Royal Statistical Society*: Series B, 26, 211-252.

Court, A.T. (1939). Hedonic price indexes with automotive examples. (In: *The dynamics of automobile demand.* New York: General Motors.)

Ekeland, I., Heckman, J.J. & Nesheim, L. (2004). Identification and estimation of hedonic models. *Journal of Political Economy*, 112(1), 60-109.

Epple, D. (1987). Hedonic prices and implicit markets: Estimating demand and supply functions for differentiated products. *Journal of Political Economy*, 95, 59-80.

Feenstra, R.C. (1995). Exact hedonic price indexes. *Review of Economics and Statistics*, 77(4), 634-653.

Fitzmaurice, G.M., Laird, N.M. & Ware, J.H. (2004). *Applied Longitudinal Analysis.* (Hoboken, New Jersey: John Wiley & Sons)

Gurka, M.J., Edwards, L.J., Muller, K.E. & Kupper, L.L. (2006). Extending the Box-Cox transformation to the linear mixed model. *Journal of Royal Statistical Society*: Series A, 169(2), 273-288.

Hsiao, C. (2003). *Analysis of Panel Data: Second edition*. (Cambridge: Cambridge University Press)

Lancaster, K. (1966). A new approach to consumer theory. *Journal of Political Economy*, 74, 132-157.

Littell, R.C., Milliken, G.A., Stroup, W.W., Wolfinger, R.D. & Schabenberber, O. (2006). *SAS for mixed models: Second edition*. (Cary, North Carolina: SAS Publishing)

Lucas, R. E. Jr. (1978). Asset prices in an exchange economy, *Econometrica*, 46, 1429-1445.

McCulloch, C.E., Searle, S.R. & Neuhaus, J.M. (2008). *Generalized, linear, and mixed models: Second edition*. (Hoboken, New Jersey: John Wiley & Sons)

Rosen, S. (1974). Hedonic prices and implicit markets: Product differentiation in pure competition. *Journal of Political Economy*, 82, 34-55.

Tables

Table 1: Summary of J-REIT pricing data.

We defined eight strata of real estate. Regarding use, we specified the four strata of *Office*, *Commercial*, *Residential*, and *Other* uses, and regarding district specified the two strata of the *three central wards of Tokyo* (*Tokyo 3*) and *all remaining districts (Other)*. These eight strata and their abbreviations are used in the following Tables from 2 to 8. We show the geometric mean prices for each of real estate strata for each year. Also we report the number of observations in parentheses.

Year	Office		Commercial		Residential		Other		Cross-Section Mean Price
	Tokyo 3	Other	Tokyo 3	Other	Tokyo 3	Other	Tokyo 3	Other	
2005	44.8433	32.9631	20.3819	54.0523	14.8630	8.0656	25.5651	23.4576	13.9551
	(38)	(47)	(2)	(20)	(75)	(220)	(2)	(4)	(408)
2006	41.6710	26.7182	15.3000	45.0320	23.5385	7.8646	21.6104	22.5451	15.6222
	(70)	(102)	(1)	(38)	(45)	(279)	(3)	(23)	(561)
2007	32.7557	34.9048	33.9411	39.5926	20.9478	9.4744	25.9000	16.2678	14.6964
	(32)	(65)	(2)	(28)	(33)	(284)	(1)	(11)	(456)
2008	34.4034	35.2773	34.0000	35.0215	20.1604	11.0089	37.9937	34.7289	20.5505
	(39)	(63)	(1)	(15)	(17)	(106)	(2)	(5)	(248)
2009	20.7946	34.3699	64.3000	28.2672	18.6143	12.4965	-	42.9212	22.1822
	(11)	(30)	(1)	(7)	(3)	(29)	(0)	(3)	(84)
2010	74.9794	40.1652	136.0000	15.9346	17.4602	9.3077	-	9.2107	11.7665
	(20)	(13)	(1)	(12)	(65)	(350)	(0)	(11)	(472)
Time-series Mean Price	40.0585	31.9882	34.8524	38.2938	18.2102	8.9680	26.5461	19.1895	14.8684

Table 2: Summary of J-REIT record summary of attribute data.

We report the arithmetic means of the contents of three attributes – floor space in square meters (*Square Meters (m2)*), age of building or house in years (*Age (Yrs)*), and walking distance from nearest subway/railway station (*Walk from the Station (Min)*) – for each of the eight real estate strata. These abbreviations are used in the following Tables from 3 to 8. Also the arithmetic means of three attributes for all the strata are reported.

Means of Attributes	Office		Commercial		Residential		Other		Means for All Strata
	Tokyo 3	Other	Tokyo 3	Other	Tokyo 3	Other	Tokyo 3	Other	
Square Meters (m2)	14,946.14	11,935.58	2,840.98	27,090.45	4,325.37	2,944.87	4,459.16	9,849.54	6,995.19
Age (Yrs)	16.72	14.93	1.98	7.37	3.69	6.26	9.61	11.14	8.40
Walk from the Station (Min)	3.18	4.05	3.63	7.70	4.21	6.67	3.00	7.77	5.76

Table 3: Comparison of *mixed effect and fixed effect models* in terms of their AICs and estimated λs for each year.

	AIC		λ	
	Fixed	Mixed	Fixed	Mixed
2005	3,041.10	2,905.90	0.05	0.05
2006	4,361.80	4,239.20	-0.07	-0.09
2007	3,513.90	3,408.80	-0.13	-0.09
2008	2,099.40	2,077.80	-0.03	-0.03
2009	666.90	659.30	-0.22	-0.14
2010	3,414.40	3,213.20	-0.06	0.02

Table 4: Estimation results of intercepts by *mixed and fixed effect models*. These are represented as dummy variables of eight strata.

Stratum		2005		2006		2007		2008		2009		2010	
		Mixed	Fixed	Mixed	Fixed	Mixed	Fixed	Mixed	Fixed	Mixed	Fixed	Mixed	Fixed
Office	Tokyo 3	42.6496*	54.527*	63.2386*	64.0612*	49.1776*	59.9285*	85.1454*	76.3343*	81.8776*	102.62*	56.8194*	59.1622*
		(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)
	Other	43.8352*	46.3648*	56.1082*	57.8056*	58.5187*	58.63*	74.8833*	80.5338*	88.9244*	107.4*	43.6059*	52.0048*
		(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)
Commercial	Tokyo 3	39.6744*	39.7611*	52.5511*	51.1319*	47.069*	57.5996*	73.8836*	75.2534*	112.98*	125.1*	53.6089*	58.8668*
		(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)
	Other	55.5944*	41.9633*	62.1125*	57.2422*	53.5545*	56.5434*	72.2208*	73.8649*	94.6554*	100.69*	26.9572*	36.7419*
		(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)
Residential	Tokyo 3	34.3118*	35.3862*	53.3059*	54.9372*	43.5752*	52.2925*	72.1975*	65.9069*	85.4672*	100.21*	28.6307*	38.6876*
		(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)
	Other	28.7545*	29.7529*	35.9783*	41.7106*	34.7964*	41.0276*	47.0455*	54.6607*	71.6609*	86.2436*	26.6741*	32.953*
		(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)
Other	Tokyo 3	41.3432*	42.9083*	52.6693*	56.2097*	48.7952*	59.8986*	77.3208*	78.6149*	-	-	-	-
		(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(-)	(-)	(-)	(-)
	Other	34.0688*	39.3445*	50.4982*	55.002*	38.5641*	50.4782*	70.7534*	76.3981*	117.63*	108.24*	29.1089*	36.8355*
		(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)

Table 5: Estimation results by *mixed and fixed effect models* for 2005.

Figures in parentheses are *P*-values.

2005	Fixed Effect Model	Mixed Effect Model								
		Fixed Coef.	Sum of Fixed and Random Coefficients							
			Office		Commercial		Residential		Other	
			Tokyo 3	Other	Tokyo 3	Other	Tokyo 3	Other	Tokyo 3	Other
Square Meters (m2)	0.0006*	0.0012*	0.0013*	0.0006*	0.0013	0.0002*	0.0011*	0.0024*	0.0012	0.0012*
	(<.0001)	(0.0104)	(<.0001)	(0.0005)	(0.1786)	(0.0184)	(0.0001)	(<.0001)	(0.2083)	(0.0227)
Age (Years)	-0.4542*	-0.3481	0.0122	-0.1033	-0.3394	-0.4733	-0.3285	-0.8271*	-0.3678	-0.3573
	(<.0001)	(0.1116)	(0.9242)	(0.5715)	(0.4412)	(0.0598)	(0.2030)	(0.0002)	(0.4036)	(0.4001)
Minutes Walk from Station (Min.)	-0.1520	-0.3787*	-0.3886	-0.3838	-0.3888	-0.4592*	-0.3168	-0.3433*	-0.3796	-0.3696
	(0.2476)	(0.0484)	(0.1065)	(0.1040)	(0.1136)	(0.0444)	(0.1522)	(0.0339)	(0.1209)	(0.1270)

Table 6: Estimation results by *mixed and fixed effect models* for 2006.

Figures in parentheses are *P*-values.

2006	Fixed Effect Model	Mixed Effect Model								
		Fixed Coef.	Sum of Fixed and Random Coefficients							
			Office		Commercial		Residential		Other	
			Tokyo 3	Other	Tokyo 3	Other	Tokyo 3	Other	Tokyo 3	Other
Square Meters (m2)	0.0004*	0.0010	0.0003*	0.0004*	0.0010	0.0002*	0.001*	0.003*	0.0011	0.0008*
	(<.0001)	(0.0628)	(0.0009)	(0.0002)	(0.4214)	(0.0008)	(0.0010)	(<.0001)	(0.3353)	(0.0008)
Age (Years)	-0.2535*	-0.1903	-0.0180	-0.1119	-0.1903	0.0237	-0.4352	-0.4301*	-0.1414	-0.2195
	(<.0001)	(0.1934)	(0.8676)	(0.4562)	(0.5295)	(0.8950)	(0.1116)	(0.0015)	(0.6181)	(0.3430)
Minutes Walk from Station (Min.)	-0.7234*	-0.8147*	-1.3423*	-0.5685	-0.8147	-1.045*	-0.7970	-0.4263*	-0.8150	-0.7092
	(<.0001)	(0.0241)	(0.0227)	(0.1077)	(0.1818)	(0.0124)	(0.1281)	(0.0450)	(0.1814)	(0.1110)

Table 7: Estimation results by *mixed and fixed effect models* for 2007.

Figures in parentheses are *P*-values.

2007	Fixed Effect Model	Mixed Effect Model								
		Fixed Coef.	Sum of Fixed and Random Coefficients							
			Office		Commercial		Residential		Other	
			Tokyo 3	Other	Tokyo 3	Other	Tokyo 3	Other	Tokyo 3	Other
Square Meters (m2)	0.0003*	0.0014*	0.0012*	0.0001	0.0024*	0.0003*	0.0019*	0.0018*	0.0014	0.0018*
	(<.0001)	(0.0178)	(0.0007)	(0.0607)	(0.0442)	(0.0007)	(0.0057)	(<.0001)	(0.2505)	(0.0405)
Age (Years)	-0.2611*	-0.1578	-0.0927	-0.1959	-0.1518	-0.1301	-0.1752	-0.2094	-0.1578	-0.1498
	(0.0016)	(0.1367)	(0.4590)	(0.1285)	(0.3073)	(0.3116)	(0.2233)	(0.0563)	(0.2907)	(0.2951)
Minutes Walk from Station (Min.)	-0.1249	-0.2189	-0.2199	-0.2231	-0.2192	-0.2224	-0.2192	-0.2159	-0.2189	-0.2127
	(0.2071)	(0.0591)	(0.0684)	(0.0640)	(0.0692)	(0.0592)	(0.0689)	(0.0650)	(0.0695)	(0.0704)

Table 8: Estimation results by *mixed and fixed effect models* for 2008.

Figures in parentheses are *P*-values.

2008	Fixed Effect Model	Mixed Effect Model								
		Fixed Coef.	Sum of Fixed and Random Coefficients							
			Office		Commercial		Residential		Other	
			Tokyo 3	Other	Tokyo 3	Other	Tokyo 3	Other	Tokyo 3	Other
Square Meters (m2)	0.0002*	0.0008	0.0002*	0.0006*	0.0008	0.0004*	-0.0003	0.0033*	0.0008	0.0008
	(<.0001)	(0.1920)	(0.0079)	(0.0068)	(0.5874)	(0.0167)	(0.3303)	(0.0008)	(0.5823)	(0.1987)
Age (Years)	-0.39*	-0.4996	-0.8875*	-0.2078	-0.4996	-0.2211	-0.6037	-0.4610	-0.5035	-0.6127
	(0.0242)	(0.1333)	(0.0332)	(0.3565)	(0.3768)	(0.6479)	(0.2851)	(0.1526)	(0.3717)	(0.2393)
Minutes Walk from Station (Min.)	-0.3613	-0.6496	-0.6496	-0.6496	-0.6496	-0.6496	-0.6496	-0.6496	-0.6496	-0.6496
	(0.3189)	(0.1204)	(0.1204)	(0.1204)	(0.1204)	(0.1204)	(0.1204)	(0.1204)	(0.1204)	(0.1204)

Table 9: Estimation results by *mixed and fixed effect models* for 2009.

Figures in parentheses are *P*-values.

2009	Fixed Effect Model	Mixed Effect Model								
		Fixed Coef.	Sum of Fixed and Random Coefficients							
			Office		Commercial		Residential		Other	
			Tokyo 3	Other	Tokyo 3	Other	Tokyo 3	Other	Tokyo 3	Other
Square Meters (m2)	0.0011*	0.0018	0.0045*	0.0019*	0.0018	0.0008*	0.0019	0.0018*	-	-0.0002
	(<.0001)	(0.0891)	(0.0163)	(0.0042)	(0.4146)	(0.0463)	(0.3186)	(0.0028)	(-)	(0.6872)
Age (Years)	-0.4295	-0.5071	-0.5742	-0.4093	-0.5071	-0.5201	-0.5092	-0.5863	-	-0.4439
	(0.0500)	(0.0725)	(0.0990)	(0.1021)	(0.1562)	(0.1009)	(0.1546)	(0.0884)	(-)	(0.1910)
Minutes Walk from Station (Min.)	-1.3077*	-1.3309	-1.3309	-1.3309	-1.3309	-1.3309	-1.3309	-1.3309	-	-1.3309
	(0.0206)	(0.0785)	(0.0785)	(0.0785)	(0.0785)	(0.0785)	(0.0785)	(0.0785)	(-)	(0.0785)

Table 10: Estimation results by *mixed and fixed effect models* for 2010.

Figures in parentheses are *P*-values.

2010	Fixed Effect Model	Mixed Effect Model								
		Fixed Coef.	Sum of Fixed and Random Coefficients							
			Office		Commercial		Residential		Other	
			Tokyo 3	Other	Tokyo 3	Other	Tokyo 3	Other	Tokyo 3	Other
Square Meters (m2)	0.0001*	0.0007	0.0000	0.0006*	0.0007	0.0001	0.0015*	0.0015*	-	0.0006
	(0.0006)	(0.0618)	(0.7190)	(0.0118)	(0.3811)	(0.0502)	(0.0004)	(<.0001)	(-)	(0.3296)
Age (Years)	-0.5027*	-0.4138*	-0.2891*	-0.4262*	-0.4138*	-0.4446*	-0.3926*	-0.4717*	-	-0.4583*
	(<.0001)	(0.0070)	(0.0350)	(0.0264)	(0.0487)	(0.0282)	(0.0496)	(0.0005)	(-)	(0.0240)
Minutes Walk from Station (Min.)	-0.1147	-0.0436	-0.1964	-0.0845	-0.0436	0.2290	0.0507	-0.2722*	-	0.0117
	(0.1919)	(0.7966)	(0.5375)	(0.7808)	(0.8955)	(0.2886)	(0.8352)	(0.0358)	(-)	(0.9295)