Time Series Modeling of Real Estate Prices and Its Application

by

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Abstract

As real estate and financial asset markets are merging in these days, there is a strong need for us to have a theoretical foundation for analysis of real estate investments in conjunction with both domestic and international financial investments. The purpose of this paper is to present a methodology to evaluate values of pieces of real estate. In particular, we extend a classical hedonic model to a sophisticated one that allows us to generate "implied" capital returns on real estate and to estimate risks and returns on real estate investments. The results of our theory and statistical analysis here highlight the role of real estate investments, contrasting to that of financial ones.

Keywords: real estate, price and rate of return, time series model, empirical analysis, financial theory.

1 Introduction

As real estate and financial asset markets are merging in these days, they together become significant driving forces of the global economy. It is widely known that the Lehman Shock and subsequent world-wide financial crisis were triggered by the crush of structured products constructed on the US residential loans. This incident called for a more profound understanding of potential returns and risks of financial investments on real estate markets. In particular, when investors appraise their financial investment opportunities on pieces of real estate, they should have an easy access to some measures of return-on-investment for these opportunities in the form comparable to financial securities. However, this is in fact far from the reality. Let us elaborate this point below.

In financial theory, especially in portfolio management theory, the characteristics of financial assets are described by two measures called "expected returns" and "risks". The former is represented by the expected ratios of total gains to asset prices over a certain period of time while the latter is represented by variances of these ratios over the same period. For stocks, bonds, currencies, and commodities traded in the exchanges, data sets of time varying market prices are usually available in every hour, or even every minute. The availability of these data sets allows the investors to update their evaluations for returns and risks of these assets in every minute.

Each piece of real estate, on the other hands, is traded in a completely different time horizon. They are typically traded once in several years, or even in a few decades. Also, there is no centralized marketplace for real estate, which makes investors face difficulties in gathering "market data" for pieces of real estate in a comprehensive way. In some countries, official land prices are estimated and announced by some governmental bodies every year. However, these prices are not actual market prices; rather they are hypothetical prices for presumed transactions.

All these facts make it difficult for investors of real estate markets to estimate expected returns and risks on their investments on the same terms with those for financial securities.

The purpose of this paper is to develop a methodology that helps to assess real estate investment decisions in the form comparable to financial securities. To this end, we propose an extended hedonic model of real estate prices on the basis of a dynamic portfolio optimization strategy with the consideration of not only financial securities but also hypothetical real estate markets. The model facilitates us to generate time series data of "implied" capital returns, from which we are able to evaluate expected returns and risks on real estate investments to conduct the mean-variance analysis for these investment decisions. We examine the market of Japanese housing apartments to demonstrate how our methodology works.

The paper is organized as follows: The next section develops the theory for financial-real estate market equilibrium, extending a classical hedonic model of real estate prices. Based on the theory, a technique for generating time series data of implied capital returns on real estate investment is presented. In Section 3, we apply the technique to a real-world data set, Japanese housing apartment transaction record. In Section 4, we discuss the implications of the obtained implied capital returns, employing the mean-variance analysis to identify the position of real estate investment vis-à-vis financial securities including domestic / foreign stocks and bonds, gold, and other commodities. Section 5 concludes our discussion.

2 Time series modeling of real estate prices and returns

This section summarizes a theory that provides a basis for statistical analysis of real estate markets.

2.1 Theory of real estate prices

Ishijima and Maeda (2012) proposed a dynamic general equilibrium model of the economy in which the three types of markets exist. These are financial security markets, real estate property markets, and real estate lease markets. Their model leads to a price formula for real estate properties as follows:

$$\begin{aligned}(\text{real estate property price}) \;&=\; \sum_k (\text{price of real estate } k\text{-th attribute}) \\ &\qquad \times (\text{quantity of real estate } k\text{-th attribute})\end{aligned} \tag{1}$$

This equation states that the equilibrium prices of real estate properties are represented as linear combinations of "attribute" prices. This is in fact an extension of a classical model of Lancaster (1966) and Rosen (1974), called "hedonic model". One of claims by Ishijima and Maeda (2012) is that in the equilibrium, though the rent of a piece of real estate is represented by a form of hedonic model, not necessarily for the price of real estate property; Ishijima and Maeda identified two technical conditions for the the price of real estate property to have the hedonic model features as in Eq. (1).

2.2 Theory of real estate returns

The dynamic equilibrium model of Ishijima and Maeda (2012) provides a theoretical foundation for a decomposition of factors that drive returns on real estate property prices. They claim that these returns are decomposed into three factors as follows:

$$\begin{aligned}&(\text{return on the real estate property price})\\&= (\text{market factor common to all the real estate})\\&\quad + (\text{idiosyncratic factor reflecting individuality for each real estate})\\&\quad + (\text{random fluctuation term as time series data})\end{aligned} \tag{2}$$

It is in fact closely related to a real estate market index known as the repeated sales model (or, weighted repeated sales index) proposed by Case and Shiller (1989). While Case and Schiller proposed their index model in an intuitive way without any theoretical foundations, Ishijima and Maeda constructed Eq. (2) on the basis of their theory. The appendix discusses the detail.

2.3 Statistical model of real estate price

Another outcome of Ishijima and Maeda (2012) is forms of statistical models that are directly applicable to empirical analysis of real estate market data. Suppose that there are M pieces of real estate in the market. They are stratified into N "strata" by areas and uses. The stratum i contains n_i pieces of real estate. It is assumed that $\sum_{i=1}^{N} n_i = M$. Each piece of real estate is characterized by K "attributes".

Let $H_{ij,t}$ denote the price of j-th real estate in i-th stratum traded at time t. Also, let $x_{ij,t}^{(k)}$ denote the amont of k-th attribute that is contained in j-th real estate in i-th stratum traded at time t. Ishijima and Maeda (2012) proposed the following statistical models:

(Fixed Effects Model)

$$H^*_{ij,t} = \alpha_t + \sum_{k=1}^{K} \beta_t^{(k)} x_{ij,t}^{(k)} + \varepsilon_{ij,t} \tag{3}$$

(Mixed Effects Model)

$$H^*_{ij,t} = \alpha_t + \sum_{k=1}^{K} \left(\beta_t^{(k)} + \nu_{i,t}^{(k)}\right) x_{ij,t}^{(k)} + \varepsilon_{ij,t} \tag{4}$$

$$\text{where } H^*_{ij,t} = \begin{cases} \frac{H^{\lambda}_{ij,t}-1}{\lambda} & (\text{if } \lambda \neq 0) \\ \log H_{ij,t} & (\text{if } \lambda = 0) \end{cases} \tag{5}$$

Note that $i = 1, \ldots, N;\ j = 1, \ldots, n_i$.

These models have two features. One is that the Box-Cox transformation (Box and Cox: 1964) is employed as Eq. (5). The other is that the idea of "mixed effects model" is applied, explicitly in Eq. (4) and implicitly in both Eqs. (3) and (4). Let us take a close look at each by each.

(1) the Box-Cox transformation

As Eq. (1) claims, the price of a piece of real estate must be a linear combination of attribute prices in theory. To follow this principle, however, the market must satisfy certain strict conditions. The real world will be much more loose than the theory expects. It is because there may exist the lack of liquidity, huge transaction costs, information asymmetry, etc., and thus, the real estate markets are distorted, diverging from a competitive-market situation.

To accommodate the divergence, the use of the Box-Cox transformation is helpful. In Eq. (5), λ denotes the transformation parameter. In the case of $\lambda = 1$, the real estate price is considered to follow a linear form. In the case λ is not equal to unity, on the other hand, there may exist a divergence from the linear real estate price.

Notice that in the literature of real estate economics, it is popular to employ log-linear hedonic regression, which corresponds to the case of $\lambda = 0$. In this sense, the use of the Box-Cox transformation gives us a higher degree of flexibility in regression analysis.

(2) Mixed effects model features

Pieces of real estate are completely different from commodities in that there is no other piece that has an identical composition of attributes. Among some real estate groups, however, pieces in each group share a similar composition of attributes, and thus their price formation can be similar. In fact, we can stratify pieces of real estate by attributs such as location and usage. We call these groups *real estate strata*. These strata may bring premia to each piece of real estate.

To include the difference that comes from individuality and classification by strata into our modeling framework, we can take two approaches. The first one is to replace the intercept of typical regression analysis with a linear combination of dummy variables of strata. In Eqs. (3) and (4), we can introduce the following replacement:

$$\alpha_t := \sum_{l=1}^{K} \beta_t^{(l)} x_{ij,t}^{(l)} \tag{6}$$

$x_{ij,t}^{(l)}$ $(l = 1, \ldots, N)$ corresponds to real estate strata, and are set such that $x_{ij,t}^{(l)} = 1$ (if $l = i$), $x_{ij,t}^{(l)} = 0$ (if $l \neq i$). As was discussed above, α_t can be interpreted as the *stratum premium* with this setting.

The second approach is to include the difference into the coefficients of typical regression analysis: A typical regression model has the form of Eq. (3). Thus, this approach is to replace $\beta_t^{(k)}$ with randomly varying coefficients. More specifically, we introduce $\nu_{i,t}^{(k)}$ that represents random variations of the fixed coefficient $\beta_t^{(k)}$ in Eq. (3) to have Eq. (4).

$\varepsilon_{ij,t}$ in Eq. (4) represents an error term that follows a M-dimension normal distribution with the mean of zero. The covariance matrix for the error term is diagonal with its element being the same. Also, $\boldsymbol{\nu}_{i,t} := \left(\nu_{i,t}^{(1)} \ldots \nu_{i,t}^{(k)} \ldots \nu_{i,t}^{(K)}\right)'$ is independent of $\varepsilon_{ij,t}$, and follows a K-dimension normal distribution with the mean of zero. The covariance matrix is denoted by $\mathbf{G}$.

The use of mixed effects models is becoming popular in statistical studies, especially in the fields of longitudinal and panel data analyses. Examples include Hsiao (2003), Fitzmaurice et al. (2004), McCulloch et al. (2008), etc.

To conduct the analysis for Eq. (4), we used the MIXED procedure of SAS 9.1.3 that was developed owing to results of Littell et al. (2006). For both models of (3) and (4) with (5), parameters are estimated by the REML (Restricted Maximum Likelihood) method as BLUP (Best Linear Unbiased Prediction). Though the covariance matrix $\mathbf{G}$ in Eq. (4) can be designed arbitrarily in general in the mixed effects model, we adopted a simplest one, that is, a diagonal matrix.

For each time t $(t = 1, \ldots, T)$, using Eqs. (3) or (4) with the Box-Cox transformation of

(5), we estimate coefficients and λs. Then, we can obtain the estimated real estate prices as follows:

$$\hat{H}_t = \begin{cases} \left(1 + \hat{\lambda} \cdot \hat{H}_t^*\right)^{1/\hat{\lambda}} & (\text{if } \hat{\lambda} \neq 0) \\ \exp\left(\hat{H}_t^*\right) & (\text{if } \hat{\lambda} = 0) \end{cases} \quad (t = 1, \ldots, T) \tag{7}$$

2.4 Implied capital return of real estate and its time series modeling

The capital returns of investing on a piece of real estate is defined as

$$\tilde{R}_{ij,t} := (H_{ij,t} - H_{ij,t-1}) / H_{ij,t-1}.$$

To estimate these capital returns at each moment t, it is enough to observe two consecutive prices of the piece of real estate at every moment, $H_{ij,t-1}$ and $H_{ij,t}$. In stock, bond, and other financial markets, this requirement is easily satisfied. However, for the market of real estate, it is not the case: As we discussed in Introduction, observing two consecutive prices is usually impossible.

To solve this difficulty, we make use of the results of estimation by Eqs. (3), (4), and (5): Since the estimation results provide the relations between $H_{ij,t}$ and the set of attributes $\boldsymbol{x}_{ij,t}$, we can approximate $H_{ij,t-1}$ by assuming we have had the same set of attributes at the time of $t-1$.

We replace $H_{ij,t-1}$ with $\hat{H}_{t-1}(\boldsymbol{x}_{ij,t})$ to approximate true capital returns $\tilde{R}$. We call it "implied capital returns" and write R as follows:

$$R_{ij,t} = \frac{H_{ij,t} - \hat{H}_{t-1}(\boldsymbol{x}_{ij,t})}{\hat{H}_{t-1}(\boldsymbol{x}_{ij,t})} \approx \tilde{R}_{ij,t} \tag{8}$$

By the use of Eq. (8), we can generate implied capital returns for any individual piece of real estate. Having the complete set of time series data, we can apply any techniques that are popular in financial economic and/or engineering analyses. Also, by the theory that we discussed in section 2.2, we can apply the decomposition of Eq. (2) to the set of implied capital returns. Using a variant of mixed effects models, we obtain the following:

$$R_{ij,t} = m_t + \mu_{i,t} + \eta_{ij,t} \tag{9}$$

$$(i = 1, \dots, N;\ j = 1, \dots, n_i)$$

Note that m_t is the market factor which is common to all the real estate and modeled as the fixed effects term. $\mu_{i,t}$ is the idiosyncratic factor which may vary according to each of strata reflecting individuality for each real estate and modeled as the mixed effect term. It follows a N-dimensional normal distribution with the mean of zero. Its diagonal covariance matrix is denoted as $\mathbf{H}$. $\eta_{ij,t}$ is the error term that follows a M-dimensional normal distribution with the mean of zero.

3 Empirical analysis

3.1 Data

We conduct an empirical analysis on Japanese housing apartment markets to apply our framework. The data sets used in our analysis consist of transaction prices and attributes for each piece of Japanese housing apartments and are obtained from the Land General Information System of Japanese Ministry of Land, Infrastructure, Transport and Tourism which are available on the Internet. We quoted data sets quarterly from the second quarter in 2006 to the fourth quarter in 2011 which amounts to twenty data sets. We focused on six areas when sampling data sets which are namely, Sapporo city, five central cities in Tokyo (hereafter Tokyo 5. These are Chiyoda, Chuo, Minato, Shibuya, Shinjuku), other 18 cities in Tokyo (Tokyo 18), Nagoya city, Osaka city and Fukuoka city. We assume that each of these six areas is assigned to one of real estate strata which reflects individuality. As dependent variables, we use the unit real estate price per square meters. And as independent variables, we adopt the age measured in years and the walking distance from the nearest railway/subway stations measured in minutes.

We summarize the average unit prices of housing apartment in Table 1. We describe the average price behavior of the Japanese apartment market in the sample period. The market experiences the bottom at the second quarter in 2006, then from the first quarter in 2007 to the first quarter in 2008 it stays in the high price range. Afterwards the real estate price goes down since the late 2008 due to the financial crisis. After the price reaches the bottom in the first quarter in 2009, it moves upward until recent days. Next we report the average price movements for each of six area strata. The rankings of six areas in order of their average prices from high to low are Tokyo 5, Tokyo 18, Osaka, Nagoya, Fukuoka and Sapporo. We discern two patterns in time series of apartment prices. The first pattern can be seen in the time series of Tokyo 5 and Tokyo 18. This is very similar to the average price movements in the whole housing apartment market, except subtle differences in timing when to reach the top or bottom. While, the second pattern can be seen in those of Sapporo, Nagoya, Osaka and Fukuoka and it seems to be almost flat. These phenomena can be understood in that since much of data belongs to the first pattern, the first and second patterns aggregate to the price movements in the whole market which is similar to the first one.

As a reference, we also show the average attributes both for the whole apartment market and each of six area strata in Table 2. Concerning the average square meters, Tokyo 5 has the smallest while Sapporo has the largest. And we can see that apartments in Tokyo are relatively young and close to the railway/subway stations.

3.2 Estimation of real estate prices

For each quarter from the second quarter in 2006 to the first quarter in 2011, we estimated unit prices of apartments by the fixed effects model (3) and mixed effects model (4). The results are shown in Table 3. We find in every quarter that the mixed effects model has a better goodness of fit than the fixed one in the sence of AICs (Akaike Information Criteria). Hence we will conduct analyses by mixed effects model (4) hereafter. By the mixed effects model, we estimated the degree of distortion in prices λ for each quarter by employing the method of Gurka et al. (2006). If λ equals to unity, the price is linear as in the theory, otherwise there may exist distortion in prices. Especially if λ equals to zero, the market implies log prices. Estimated λ by mixed effects model takes the small positive values ranged from 0.06 to 0.29. Hence we can say that there exists distortion in prices which is close to the log price but slightly towards the linear price.

We also estimated the intercept as well as the two factors of attributes, that are age and walking distance from the nearest railway/subway stations. These are significant for all data sets in every area stratum and in every quarter except a few case in Tokyo 5. We can then conclude that the mixed effects model is able to estimate the Japanese apartment prices significantly by two factors and intercepts.

3.3 Generating implied capital returns

By utilizing the estimated mixed effects model of Eq. (4) which provides better goodness of fit to data, we are able to generate the implied capital returns as we discussed in section 2.4. Since we estimated the model for each of 20 quarterly data sets from the second quarter in 2006 to the first quarter in 2011, 19 implied capital returns are quarterly available. For these implied capital returns, we summarize their means and standard deviations for the whole market and for each of six area strata at every quarter, as shown in Table 4.

We can see that the behavior of return averages for the whole market corresponds to that of price averages which is discussed in section 3.1. With a series of negative returns in mid-2006, the market experiences the bottom, then from the first quarter in 2007 to the first quarter in 2008 it stays in high price range owing to a series of positive returns. Afterwards negative returns cause real estate prices to go down since the late 2008 due to the financial crisis. After the price reaches the bottom in the first quarter in 2009, it moves upward until recent days with a series of positive returns.

Next, we take a look at tendencies of the return behavior for each of six area strata. Nagoya and Fukuoka have high-risk with highest-return profiles. While Tokyo 18 and Sapporo acquire the second high-return with lower-risk. Tokyo 5 and Osaka have lowest-return, but not accompanying lowest-risk. Hence their performances result in lowest return-to-risk ratios. We find that risk-return profiles of implied capital returns are different from each other among six area strata.

3.4 Decomposing implied capital returns to make return indices

By using our time-series model of (9), we decompose quarterly-estimated implied capital returns into the following: the market factor that is common to all apartments (m_t), the idiosyncratic factor that may vary according to each of six area strata reflecting individuality for each apartment ($\mu_{i,t}$), and the error term ($\eta_{ij,t}$). In fact, we estimated the model (9) as a mixed effects model, making use of MIXED procedure of SAS 9.1.3. We show the estimated results in Table 5. As a result, for each of six area strata, the decomposed idiosyncratic factor $\mu_{i,t}$ turned out to take almost the same value as the average returns at every quarter, as shown in Table 4. For the whole market, however, the decomposed market factor m_t does not match the average returns as shown in Table 4. The latter can be merely interpreted as the statistical average. While decomposed market and idiosyncratic factors can gain the reasonable interpretation since they are estimated by the valid time series model (9) which is elaborated from the theory regarding (2). Hence the estimated market factor m_t can be viewed as a return index which serves as a benchmark for the whole apartment market. Since this return index is observable along the constant time-interval, we can directly apply financial theory. That is, we are now in the position to analyze the real estate investment through financial theory vis-à-vis other financial securities.

4 Applications of the return index: The importance of real estate in asset allocation

In the previous section, we elaborate the framework on how to generate the time series of implied capital returns, how to model these with reasonable foundation, and how to obtain the return index of the whole real estate market along the constant time-interval. We are now in position to apply financial theory to this return index directly. Hence our framework has the meanings in that we merged real estate into the financial assets to which we can apply the financial theory. As one of those applications, we conduct an empirical analysis whether real estate investment is favorable from the viewpoint of its risk-return profile vis-à-vis financial securities. As target investment, in addition to traditional four assets of domestic and foreign stocks and bonds, we include so-called alternative assets. Here alternative assets have not been considered to be included in an investment portfolio and considered to be low correlated with traditional assets. For example, these are hedge funds, private equity, commodities, emissions trading, etc. Also real estate is a typical alternative asset. As alternative assets to be included in our portfolio, we select hedge funds and gold since they have enough track records and perform well these days.

Sample period is 19 quarters from the third quarter in 2006 to the first quarter in 2011 which is the same as in the previous analysis. For above-mentioned assets, we use the benchmarks as shown below: We use MSCI Japan Net Index (in yen) for domestic stocks, Nomura BPI (Total Index) for domestic bonds, MSCI Kokusai Net Index (in US dollar but converted to yen) for foreign stocks, Citigroup Non JPY WGBI (in yen) for foreign bonds, and HFRX Global Hedge Fund Index for hedge funds. As the benchmark of gold, we use its spot prices per troy ounce in dollar. We retrieve the data from Bloomberg with the ticker code “GOLDS Cmdty” and convert to yen.

For seven assets which consist of six assets plus housing apartments, we show in Table 6 the averages, standard deviations and the return-to-risk ratios which are the averages divided by standard deviations. Also on the risk-return plane in which we take risk or standard deviation on x-axis and return or average on y-axis, we show the risk-return profile by square markers for each of seven assets on Figure 1. From this figure, we can say that domestic bonds have the lowest risk while both foreign and domestic stocks have the highest risk. The assets with middle risk are shown to be gold, hedge fund, foreign bonds and apartments. Quoted from the classical Markowitz mean-variance model (for example, Luenberger, 1997), one should make a portfolio selection according to two criteria:

Criterion 1: Minimize the portfolio risk subject to the target return

Criterion 2: Maximize the portfolio return subject to the target risk

According to these two criteria, the risk-return profile should be plotted on the first quadrant. More Specifically, it should be plotted on the southwest area for assets with low-risk and low-return, the northeast area for assets with high-risk and high-return, and the area between for assets with middle-risk and middle-return. Since foreign and domestic stocks and foreign bonds are located in the fourth quadrant, they seem to be unfavorable investments in the sense of the mean-variance model. While hedge funds should be selected when compared to foreign stocks from Criterion 2, it should not be when compared to apartments from Criterion 1. Hence in the sense of mean-variance model, we should invest only in domestic bonds, gold and apartments. This consideration is coincide with the rankings concerning return-to-risk ratios in Table 6.

Next we proceed to consider the optimal mean-variance portfolio which we should select. The portfolio can be characterized by the portfolio weights or the ratios of how much funds we should invest in each asset to the total investment funds. We are to select the optimal weights according to the mean-variance model. More specifically, we solve a quadratic programming problem which Criterion 1 is reduced to be and obtain the optimal weights to each of seven assets.

By varying the target return of Criterion 1 within the possible range, we obtain the pairs of risk and return profiles of optimal portfolios to constitute the minimum variance set. This is shown in Figure 1 as the curve which connects gold and domestic stocks. Among the minimum variance set, the one with lowest risk is called a globally minimum variance portfolio which appears as MVP in Figure 1. Simultaneously, according to Criterion 2, only the upper curve of minimum variance set above MVP – that is the solid line which connects MVP and gold – will be selected. This is called a efficient frontier. We mention the optimal portfolio obtained.

MVP invests more than 90% in domestic bonds and apartments: the detailed composition is 76.6% in domestic bonds and 15.5% in apartments. This result shows that if one aims to reduce risk more than domestic bonds having the lowest risk, one has to include apartments which have strongly negative correlations with domestic bonds as shown in Table 6. While if one tries to take return regardless of risk more than MVP, one has to invest more to gold which has strongly negative correlations with domestic bonds as shown in Table 6. The highest return can be obtained by the portfolio which consists of 100% gold and located in the highest end of the efficient frontier.

From above analysis, we find that apartments have the middle risk-return profile between domestic bonds and gold and very important asset in the aspect of mean-variance model. These findings give an implication for institutional investors: It is very favorable for them to incorporate apartments in their portfolios from the viewpoints of risk-return profile. In addition to apartments, they only have to include domestic bonds or gold in their portfolios according to their risk tolerance. Our results at the beginning of 2010s can be obtained exclusively when we apply our return index of real estate – which is available from our framework – to asset allocation problems.

5 Conclusions

This paper presented a methodology for generating “implied” capital returns on real estate investment. It allows us to conduct the mean-variance analysis to indentify the positions of real estate investments vis-à-vis financial securities. The significance and contributions of this paper are summarized as follows: First, we developed a theory of financial-real estate market dynamic equilibrium. Second, on the basis of the theory, we presented a statistical model to be used for the analysis of real estate market prices and a time series model that helps to describe capital returns on real estate investments. Thirdly, we elaborated the generation of implied capital returns on real estate investments. Fourth, we demonstrated empirical analysis on the Japanese housing apartment market. Finally, applying the mean-variance analysis to the obtained implied capital returns, we mapped the position of Japanese housing apartment investment market on the mean-variance plane and discussed the potential benefit of the investment in contrast to other financial securities.

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Appendix A Derivation of a theoretical model of real estate returns

This appendix elaborates the derivation of Eq. (2) and the theory behind that is developed by Ishijima and Maeda (2012).

We consider an economy in which two types of assets are traded, financial assets N^P and real estate assets N^H, and in which economic activities are represented by the trade and consumption of a representative agent whose goal is to dynamically maximize her utility. Although a piece of real estate is a type of asset, it differs from financial assets in that merely holding the property rights of a piece of real estate does not necessarily imply that it will yield a return on investment, as it is not until a piece of real estate is utilized that it yields returns or “benefits.” To clarify this matter, we propose the following assumption:

Assumption 1

To obtain cash flows from a piece of real estate, an owner must rent it out for the duration for which she holds the property rights to it. A lessee or tenant of the property pays the owner a rent which represents the economic value of the benefit that the tenant gains from the use of that property. The benefit derived from using this piece of real estate accrues entirely to the tenant as its user.

This assumption, in particular, the last sentence implies that for the owner to enjoy the benefits accruing from the use of a piece of real estate, she would need to rent the property to herself via a “pseudo” payment of the rent referred to as imputed rent. It is noteworthy that the owner and tenant are identical when we assume the existence of a representative agent in the economy. This assumption also indicates the possibility of two types of trade for a piece of real estate: trade for its property rights and trade for the right to use it by leasing the property. Thus, we hereafter consider that three types of markets exist within the economy: financial security markets, real estate property markets, and real estate lease markets.

As discussed above, a piece of real estate provides “benefits” to its users, the values of which are reflected in the rents paid by the tenants to the owners. To convert these benefits into economic values, we introduce the concept of attributes based on the following assumption:

Assumption 2

Each piece of real estate is a representation of a bundle of attributes.

The solution for the representative agent’s dynamic optimization problem together with market clearing conditions leads to a competitive equilibrium that satisfies the following set of equations:

$$P_{j,t} = E_t\left[\left(P_{j,t+1} + D^P_{j,t+1}\right)\tilde{M}^C_{t+1}\right] \ (j = 1, \ldots, N^P) \tag{10}$$

$$H_{i,t} = L_{i,t}D^H_{i,t} + E_t\left[H_{i,t+1}\tilde{M}^C_{t+1}\right] \ (i = 1, \ldots, N^H) \tag{11}$$

$$D^H_{i,t} = \boldsymbol{b}_{i,t}\tilde{\boldsymbol{M}}^Z_t \ (i = 1, \ldots, N^H) \tag{12}$$

Notations are defined as follows:

$(P_{j,t})$: Financial security price vector at time t

$(D^P_{j,t})$: Vector of dividends yielded by financial securities at time t

$\tilde{M}^C_{t+1} := \delta \cdot \partial u(C_{t+1}, \boldsymbol{Z}_{t+1})/\partial C_{t+1} \Big/ \partial u(C_t, \boldsymbol{Z}_t)/\partial C_t$: discount factors of the products of marginal rates of substitution between consumption during two consecutive periods

u: temporal utility as a function of C and Z

δ: time preference

C_t: Representative agent's consumption at time t

$Z_{j,t}$: Amount of attribute j that is contained in a portfolio of real estate in use at time t

$(H_{i,t})$: Real estate price vector at time t

$(D^H_{i,t})$: Vector of rents paid by lessees to lessors at time t

$L_{i,t}$: Occupancy rate of piece of real estate i at time t

$b_{ij,t}$: Unit content of attribute j that is contained in real estate i at time t

$\tilde{M}^Z_{k,t} := \partial u(C_t, \boldsymbol{Z}_t)/\partial Z_{k,t} \Big/ \partial u(C_t, \boldsymbol{Z}_t)/\partial C_t \ (k = 1, \ldots, K)$: the marginal rates of substitution between the attributes and consumption

Consider a steady state in which all variables in Eqs. (10), (11), and (12) are time-invariant. In this steady state, marginal rates of substitution between consumption during two consecutive periods ($\tilde{M}^C_{t+1}$) are equalized to δ. That is, $\tilde{M}^C_{t+1} = \delta$. With this condition, Eqs. (10) and (11) lead to the following equality:

$$\left(\frac{L_i D^H_i}{H_i}\right)^{-1} = 1 + \left(\frac{D^P_j}{P_j}\right)^{-1} \quad \forall i, j \ . \tag{13}$$

Using Eq. (11) recursively, we can write real estate prices in equilibrium as follows:

$$H_{i,t} = \sum_{\tau=0}^{\infty} E_t\left[\delta^\tau L_{i,t+\tau} \boldsymbol{b}_{i,t+\tau} \boldsymbol{M}^Z_{t+\tau}\right] \tag{14}$$

where $\boldsymbol{M}^Z_{t+\tau} := \partial u\left(C_{t+\tau}, \boldsymbol{Z}_{t+\tau}\right)/\partial \boldsymbol{Z}_{t+\tau} \Big/ \partial u\left(C_{t+\tau}, \boldsymbol{Z}_{t+\tau}\right)/\partial C_t$.

For Eq. (14) to reflect the hedonic property, we introduce another assumption:

Assumption 3

The contents of the attributes contained in each piece of real estate remain constant over time such that

$$\boldsymbol{b}_{i,t} = \boldsymbol{b}_i \quad \forall i, t \tag{15}$$

This assumption accounts for many attributes, including floor space in square meters and walking distance from the nearest subway/railway stations. However, for some attributes, such as age of building or house in years, the assumption may not apply, and thus its application may be limited. Despite this limitation, application of this assumption yields insightful results when it is applied to Eq. (14) to have the following:

$$H_{i,t} = \boldsymbol{b}_i E_t \left[\sum_{\tau=0}^{\infty} \delta^{\tau} L_{i,t+\tau} \boldsymbol{M}_{t+\tau}^{Z} \right] \tag{16}$$

This is equivalent to:

$$\begin{aligned} H_{i,t+1} &= \boldsymbol{b}_i \delta^{-1} \sum_{\tau=0}^{\infty} \delta^{\tau+1} E_{t+1} \left[L_{i,t+\tau+1} \boldsymbol{M}_{t+\tau+1}^{Z} \right] \\ &= \delta^{-1} \boldsymbol{b}_i \sum_{k=1}^{\infty} \delta^{k} E_{t+1} \left[L_{i,t+k} \boldsymbol{M}_{t+k}^{Z} \right] \\ &= \delta^{-1} \boldsymbol{b}_i \sum_{k=0}^{\infty} \delta^{k} E_{t+1} \left[L_{i,t+k} \boldsymbol{M}_{t+k}^{Z} \right] - \delta^{-1} \boldsymbol{b}_i E_{t+1} \left[L_{i,t} \boldsymbol{M}_{t}^{Z} \right] . \end{aligned} \tag{17}$$

By taking expectations at time t, we obtain:

$$E_t \left[H_{i,t+1} \right] = \delta^{-1} \boldsymbol{b}_i \sum_{k=0}^{\infty} \delta^{k} E_t \left[L_{i,t+k} \boldsymbol{M}_{t+k}^{Z} \right] - \delta^{-1} \boldsymbol{b}_i L_{i,t} \boldsymbol{M}_{t}^{Z} = \delta^{-1} H_{i,t} - \delta^{-1} \boldsymbol{b}_i L_{i,t} \boldsymbol{M}_{t}^{Z} \tag{18}$$

or equivalently,

$$\frac{E_t \left[H_{i,t+1} \right] - H_{i,t}}{H_{i,t}} = \left(\delta^{-1} - 1 \right) - \frac{\delta^{-1} \boldsymbol{b}_i L_{i,t} \boldsymbol{M}_{t}^{Z}}{H_{i,t}} . \tag{19}$$

To further modify Eq. (19), recall Eq. (13). The equation holds true for all i and j in a steady state, which indicates that the market can be represented by a portfolio of assets in the equilibrium state. We call it "market portfolio", m, and write its dividend-price ratio as $r_{m,t} := D^P_{m,t}/P_{m,t-1}$. Including random variations around the steady state, $L_{i,t}D^H_{i,t}/H_{i,t} = \boldsymbol{b}_i L_{i,t}\boldsymbol{M}^Z_t/H_{i,t}$ is expressed by the use of $r_{m,t}$ as follows:

$$\frac{\boldsymbol{b}_i L_{i,t}\boldsymbol{M}^Z_t}{H_{i,t}} = \frac{r_{m,t}}{1+r_{m,t}} - \delta\sigma_i\varepsilon_{i,t} \tag{20}$$

where $\sigma_i\varepsilon_{i,t}$ represents random variations around $r_{m,t}/(1+r_{m,t})$ that are specific to real estate i. It is assumed that $\varepsilon_{i,t} \sim \mathscr{N}(0,1)$.

Assuming that $r_{m,t}$ itself follows a stochastic process, we can rewrite $r_{m,t}/(1+r_{m,t})$ as follows:

$$\frac{r_{m,t}}{1+r_{m,t}} := \hat{\mu}_{m,t-1} - \delta\sigma_{m,t-1}\varepsilon_{m,t} \tag{21}$$

where $\hat{\mu}_{m,t-1} := E_{t-1}\left[\frac{r_{m,t}}{1+r_{m,t}}\right]$, $\sigma^2_{m,t-1} := \delta^{-2}E_{t-1}\left[\left(\frac{r_{m,t}}{1+r_{m,t}} - \hat{\mu}_{m,t-1}\right)^2\right]$, and $\varepsilon_{m,t} \sim \mathscr{N}(0,1)$. It is also assumed that $\varepsilon_{i,t}$ and $\varepsilon_{m,t}$ are mutually independent.

Using Eqs. (20) and (21), we can modify Eq. (19) as follows:

$$\frac{E_t\left[H_{i,t+1}\right] - H_{i,t}}{H_{i,t}} = \mu_{m,t-1} + \sigma_{m,t-1}\varepsilon_{m,t} + \sigma_i\varepsilon_{i,t} \tag{22}$$

where $\mu_{m,t-1} := (\delta^{-1}-1) - \delta^{-1}\hat{\mu}_{m,t-1}$.

Introducing a term that represents measurement noise in time-series data, $\tilde{\eta}_{i,t+1} \sim \mathscr{N}(0,1)$, we express returns on real estate price gains $\Delta H_{i,t}/H_{i,t} := \left(H_{i,t+1} - H_{i,t}\right)/H_{i,t}$ as follows:

$$\frac{\Delta H_{i,t}}{H_{i,t}} = \mu_{m,t-1} + \sigma_{m,t-1}\varepsilon_{m,t} + \sigma_i\varepsilon_{i,t} + \sigma^\eta_{i,t}\tilde{\eta}_{i,t+1} \tag{23}$$

where $\left(\sigma^\eta_{i,t}\right)^2 := E_t\left[\left(\frac{\Delta H_{i,t}}{H_{i,t}} - E_t\left[\frac{\Delta H_{i,t}}{H_{i,t}}\right]\right)^2\right]$.

We assume that $\tilde{\eta}_{i,t+1}$, $\varepsilon_{m,t}$, and $\varepsilon_{i,t}$ are mutually independent.

Terms of the right side of Eq. (23) are interpreted as follows:

(i) The first and second terms represent random variations that are common to all pieces of real estate.

(ii) The third term represents random walk variations that are specific to individual pieces of real estate.

(iii) The fourth term represents white noise in time-series data.

It should be emphasized that first three terms are observable, or "information-measurable" at time t, while the last term is observable at time $t+1$. On the basis of the above discussion, we conclude that returns on real estate prices are decomposed to three factors as is shown in Eq. (2).

In closing this appendix, it is noteworthy that Eq. (2) is closely related to "the weighted repeated sales index" proposed by Case and Shiller (1989). The index is intuitively proposed as follows:

$$\check{P}_{i,t} = \check{C}_t + \check{H}_{i,t} + \check{N}_{i,t} \tag{24}$$

where

$\check{P}_{i,t}$: logarithm of the price of house i at time t,

$\check{C}_t$: logarithm of the average price of houses in the considered area at time t,

$\check{H}_{i,t}$: random walk term, independent of $\check{C}_t$.

$\Delta\check{H}_{i,t}$ has the mean of zero, and follows normal distribution with variance σ_h^2. $\check{N}_{i,t}$ does not have any correlation with $\check{C}_t$ and $\check{H}_{i,t}$.

Eqs. (2) and (24) are different from each other in that the former represents the changes in prices whereas the latter represents log-prices. However, they can be transformed to each other by the use of the Ito formula and some log-linear approximations (Campbell and Viceira, 2002). In this sense, Eq. (2) is interpreted as a justification for Eq. (24).

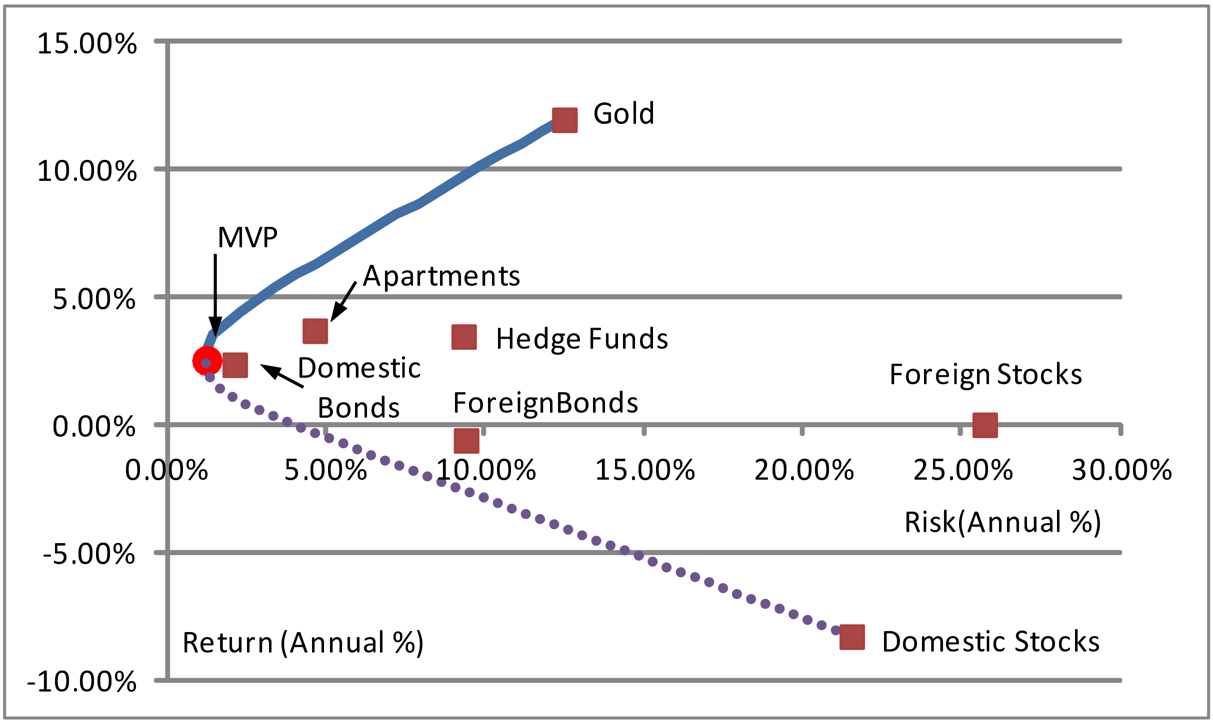

Figure 1: Risk and return profiles of traditional and alternative assets, and the efficient frontier given by mean-variance model of Markowitz (solid line).

Table 1: Quarterly reported the number of observations (N) and average prices (in ten thousand yen) for housing apartment data.

	Whole Market		Sapporo		Tokyo 5		Tokyo 18		Nagoya		Osaka		Fukuoka	
	N	Ave.	N	Ave.	N	Ave.	N	Ave.	N	Ave.	N	Ave.	N	Ave.
2006_2	1,507	48.67	84	13.92	215	81.26	674	57.81	120	24.04	287	33.16	96	25.44
2006_3	1,502	50.43	60	16.16	177	78.83	757	62.38	118	21.59	289	29.21	84	19.59
2006_4	1,572	52.92	60	15.46	199	74.73	764	66.06	156	22.28	279	34.30	97	25.93
2007_1	2,005	57.23	79	15.67	239	80.69	1,083	70.16	131	21.22	292	34.88	139	22.51
2007_2	3,313	55.33	237	15.74	457	83.58	1,652	67.44	278	27.84	447	34.72	195	20.98
2007_3	3,161	55.20	283	15.96	403	91.81	1,559	67.81	237	25.94	457	33.00	183	21.62
2007_4	3,160	56.42	309	15.37	451	87.80	1,541	69.43	244	26.01	421	37.40	150	21.37
2008_1	3,197	57.07	238	15.77	528	87.46	1,565	68.56	250	25.12	430	32.97	148	20.83
2008_2	3,391	53.60	277	15.43	531	86.77	1,590	63.94	240	23.96	473	33.97	223	22.35
2008_3	3,320	53.26	270	15.40	511	85.36	1,554	63.81	258	23.53	458	33.45	224	24.17
2008_4	3,319	50.53	300	15.81	442	79.36	1,588	62.11	244	24.83	459	32.93	214	21.66
2009_1	3,389	50.36	279	15.89	445	79.22	1,673	61.06	270	24.97	473	32.79	196	21.09
2009_2	3,693	50.95	356	15.94	587	81.73	1,672	61.19	261	25.39	549	32.24	194	21.75
2009_3	3,728	52.63	276	16.33	585	83.39	1,789	61.34	264	26.15	579	34.28	187	19.74
2009_4	3,758	52.95	295	15.28	556	81.77	1,793	64.28	263	24.85	614	33.38	191	22.12
2010_1	3,881	53.19	284	15.03	499	83.32	1,923	65.18	292	26.81	624	33.03	200	21.42
2010_2	3,782	53.27	313	14.41	526	89.17	1,836	64.29	267	28.00	569	31.02	211	20.53
2010_3	3,316	51.59	292	16.05	346	85.91	1,541	67.49	279	27.12	597	30.29	198	21.44
2010_4	2,783	52.70	257	14.68	398	91.85	1,300	63.83	178	26.11	458	30.84	143	18.88
2011_1	2,556	55.71	94	16.94	378	88.60	1,335	63.80	225	26.95	414	32.29	87	19.53

Table 2: Averages of floor space (square meters), age of apartment (years) and walking distance from the nearest subway/railway stations (minutes) for housing apartment data.

Attributes	Sapporo	Tokyo 5	Tokyo 18	Nagoya	Osaka	Fukuoka	Whole Market
Ave. Floor Space(m^2)	69.40	45.24	46.63	66.01	55.19	54.04	51.35
Ave. Age(Years)	17.10	13.17	12.30	16.07	17.27	14.39	13.96
Ave. Walk Distance(Min.)	8.43	5.46	7.60	8.74	6.24	9.80	7.38

Table 3: Comparison of AICs when housing apartment prices are estimated quarterly by fixed and mixed effects models, and the distortion coefficients λ estimated by the mixed effect model.

	AIC		
	Fixed	Mixed	Estimated λ
2006_2	12,256.72	12,194.42	0.06
2006_3	12,382.24	12,354.03	0.06
2006_4	12,993.48	12,974.37	0.22
2007_1	16,779.48	16,747.81	0.11
2007_2	27,619.17	27,534.10	0.14
2007_3	26,618.19	26,524.05	0.09
2007_4	27,005.86	26,967.65	0.13
2008_1	27,192.92	27,076.73	0.07
2008_2	28,620.04	28,494.76	0.10
2008_3	27,794.58	27,695.48	0.16
2008_4	27,533.08	27,441.95	0.22
2009_1	28,099.55	28,037.24	0.13
2009_2	30,755.18	30,589.63	0.08
2009_3	31,092.53	30,970.32	0.11
2009_4	31,532.57	31,432.84	0.15
2010_1	32,390.27	32,293.65	0.18
2010_2	32,122.08	32,062.15	0.17
2010_3	27,731.66	27,708.73	0.19
2010_4	23,031.47	22,973.59	0.15
2011_1	21,847.14	21,839.28	0.29

Table 4: Quarterly reported the averages μ and standard deviations σ for implied capital returns on housing apartment prices.

	Whole Market		Sapporo		Tokyo 5		Tokyo 18		Nagoya		Osaka		Fukuoka	
	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
2006_3	-0.28%	5.26%	2.55%	9.35%	-3.39%	2.33%	2.83%	2.23%	-7.10%	3.22%	-3.78%	6.13%	-1.89%	2.60%
2006_4	5.14%	8.03%	9.12%	1.98%	-3.43%	7.64%	5.34%	2.92%	3.23%	7.24%	5.64%	5.75%	17.99%	16.34%
2007_1	4.06%	7.14%	-7.62%	3.42%	8.93%	6.56%	4.76%	6.39%	5.93%	4.39%	0.63%	4.19%	2.73%	9.97%
2007_2	1.22%	5.72%	1.03%	3.00%	1.68%	4.64%	-1.27%	2.88%	11.47%	8.51%	6.77%	0.90%	-4.45%	2.20%
2007_3	0.87%	5.60%	-0.21%	1.74%	4.54%	3.08%	1.74%	6.12%	2.37%	7.38%	-3.65%	2.12%	-2.78%	2.50%
2007_4	0.78%	5.32%	-2.41%	3.00%	-3.26%	5.10%	1.76%	3.13%	-2.81%	3.16%	6.34%	2.84%	-0.08%	11.51%
2008_1	0.73%	7.15%	3.97%	4.33%	-2.10%	12.40%	1.63%	3.19%	2.44%	7.71%	-1.91%	7.79%	0.92%	6.96%
2008_2	-2.10%	4.74%	0.33%	1.12%	1.52%	2.58%	-4.29%	1.11%	-3.40%	4.16%	-2.94%	5.60%	3.54%	9.57%
2008_3	0.15%	2.79%	-1.31%	0.55%	0.27%	2.90%	-0.77%	1.35%	5.41%	4.94%	-0.50%	1.43%	2.70%	1.18%
2008_4	-4.10%	3.41%	-5.20%	4.25%	-5.07%	3.07%	-3.81%	0.41%	-4.50%	5.93%	-1.59%	2.85%	-6.77%	6.07%
2009_1	-1.25%	4.30%	5.50%	5.75%	-2.27%	2.57%	-2.11%	1.16%	1.81%	3.81%	-3.53%	1.56%	-0.20%	9.70%
2009_2	0.01%	4.03%	-2.59%	3.90%	0.15%	2.35%	0.35%	1.78%	0.13%	5.42%	0.01%	4.55%	0.98%	9.50%
2009_3	1.68%	3.87%	2.03%	5.88%	1.09%	1.06%	1.38%	2.39%	2.37%	2.80%	2.36%	5.02%	2.57%	8.72%
2009_4	1.04%	3.00%	-1.80%	2.12%	0.73%	2.63%	2.25%	2.07%	-1.57%	3.39%	-0.91%	2.55%	4.16%	3.29%
2010_1	3.59%	3.59%	2.93%	1.67%	4.05%	5.15%	2.95%	2.46%	6.55%	5.80%	4.62%	3.40%	2.35%	3.17%
2010_2	-0.47%	3.77%	-3.48%	2.60%	3.81%	3.95%	0.00%	1.70%	0.08%	4.44%	-3.23%	1.09%	-3.26%	6.88%
2010_3	1.08%	4.23%	4.21%	3.25%	-2.57%	1.10%	1.78%	2.32%	2.81%	8.67%	-2.66%	2.71%	5.00%	1.77%
2010_4	1.90%	4.42%	-2.81%	3.31%	3.65%	2.87%	2.46%	3.83%	2.92%	2.97%	4.35%	1.77%	-5.99%	4.54%
2011_1	0.52%	5.52%	10.68%	7.56%	1.03%	1.38%	-2.69%	3.63%	5.39%	3.87%	3.89%	2.36%	6.34%	10.15%

Table 5: Decomposition of “implied” capital returns on housing apartment prices.

	Whole Market	Sapporo	Tokyo 5	Tokyo 18	Nagoya	Osaka	Fukuoka
2006_3	-1.80%	2.48%	-3.39%	2.83%	-7.06%	-3.78%	-1.89%
2006_4	6.31%	9.09%	-3.39%	5.34%	3.25%	5.65%	17.90%
2007_1	2.58%	-7.46%	8.90%	4.75%	5.90%	0.64%	2.73%
2007_2	2.54%	1.04%	1.68%	-1.27%	11.45%	6.77%	-4.44%
2007_3	0.34%	-0.21%	4.51%	1.74%	2.35%	-3.62%	-2.74%
2007_4	-0.08%	-2.40%	-3.25%	1.76%	-2.80%	6.32%	-0.08%
2008_1	0.81%	3.87%	-2.05%	1.63%	2.38%	-1.86%	0.92%
2008_2	-0.88%	0.32%	1.51%	-4.28%	-3.39%	-2.93%	3.52%
2008_3	0.97%	-1.30%	0.27%	-0.77%	5.40%	-0.49%	2.70%
2008_4	-4.49%	-5.19%	-5.07%	-3.81%	-4.50%	-1.61%	-6.75%
2009_1	-0.13%	5.48%	-2.27%	-2.11%	1.80%	-3.52%	-0.20%
2009_2	-0.16%	-2.53%	0.14%	0.35%	0.12%	0.01%	0.93%
2009_3	1.95%	2.02%	1.16%	1.40%	2.31%	2.34%	2.48%
2009_4	0.48%	-1.79%	0.73%	2.25%	-1.56%	-0.91%	4.14%
2010_1	3.91%	2.95%	4.05%	2.95%	6.51%	4.62%	2.38%
2010_2	-1.01%	-3.47%	3.80%	-0.01%	0.07%	-3.22%	-3.25%
2010_3	1.43%	4.20%	-2.56%	1.78%	2.81%	-2.66%	4.99%
2010_4	0.76%	-2.80%	3.64%	2.46%	2.91%	4.35%	-5.97%
2011_1	4.10%	10.63%	1.04%	-2.68%	5.38%	3.89%	6.33%

Table 6: Risk (standard deviation, in annual %), return (average, in annual %), return per unit risk (return-to-risk ratio) for traditional and alternative assets, and the correlation coefficient (%) between these assets.

	Domestic Stocks	Domestic Bonds	Foreign Stocks	Foreign Bonds	Apartments	Hedge Funds	Gold
Std. Dev. (Annual %)	21.52%	2.12%	25.70%	9.39%	4.64%	9.32%	12.48%
Average (Annual %)	-8.28%	2.38%	0.02%	-0.59%	3.71%	3.48%	11.97%
Return-to-Risk Ratio	-0.38	1.12	0.00	-0.06	0.80	0.37	0.96
Correlation	Domestic Stocks	Domestic Bonds	Foreign Stocks	Foreign Bonds	Apartments	Hedge Funds	Gold
Domestic Stocks	100.00%	-55.86%	87.59%	51.31%	38.93%	85.59%	29.64%
Domestic Bonds	-55.86%	100.00%	-56.31%	-50.48%	-47.81%	-45.42%	-44.59%
Foreign Stocks	87.59%	-56.31%	100.00%	73.49%	53.47%	88.81%	43.81%
Foreign Bonds	51.31%	-50.48%	73.49%	100.00%	39.77%	49.80%	53.64%
Apartments	38.93%	-47.81%	53.47%	39.77%	100.00%	45.21%	29.25%
Hedge Funds	85.59%	-45.42%	88.81%	49.80%	45.21%	100.00%	43.70%
Gold	29.64%	-44.59%	43.81%	53.64%	29.25%	43.70%	100.00%